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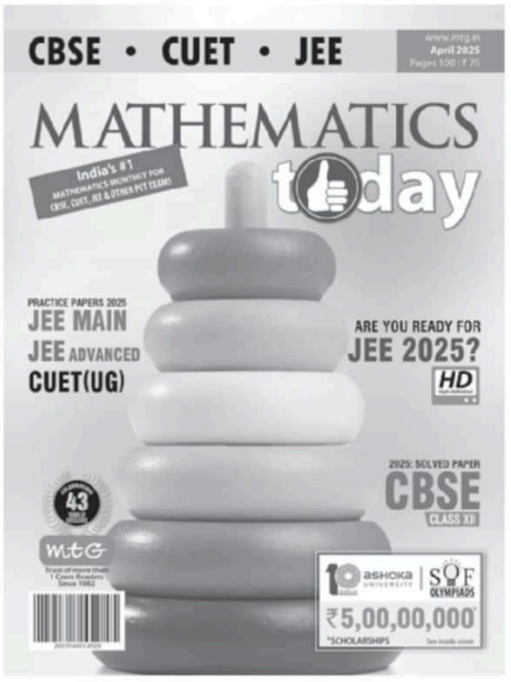

MATHEMATICS today

VOLUME 43 No. 4 April 2025

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Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat

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PRACTICE PAPER

JEE Main

2025

Exam Dates
Session-2
02, 03, 04, 07 and 08
April 2025

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. If $a \in [-6, 12]$, then the probability that graph of $y = -x^2 + 2(a+4)x - (3a+40)$ is strictly below x-axis is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) none of these

2. If $f(x) = \log_{e^2 x}\left(\frac{2\ln x + 2}{-x}\right)$ and $g(x) = \{x\}$, where $\{x\}$ denotes the fractional part of x, then the range of $g(x)$ for the existence of $f(g(x))$ is

(a) $\left(0, \frac{2}{e}\right)$ (b) $\left(0, \frac{1}{e}\right) - \left\{\frac{1}{e^2}\right\}$

(c) $\left(0, \frac{3}{e}\right)$ (d) none of these

3. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(a) 264 (b) 265 (c) 53 (d) 67

4. Let $g(x) = 2f\left(\frac{x}{2}\right) + f(2-x)$ and $f''(x) < 0$ $\forall\ x \in (0, 2)$. Then $g(x)$

(a) increases in (0,1) and decreases in (1, 2)

(b) decreases in (0,1) and increases in (1, 2)

(c) increases in $\left(0, \frac{4}{3}\right)$ and decreases in $\left(\frac{4}{3}, 2\right)$

(d) decreases in $\left(0, \frac{4}{3}\right)$ and increases in $\left(\frac{4}{3}, 2\right)$

5. If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ such that $|a_1| > |a_2| + |a_3|$, $|b_2| > |b_1| + |b_3|$, $|c_3| > |c_1| + |c_2|$, then A is

(a) orthogonal (b) singular

(c) non-singular (d) non-orthogonal

6. A particle is projected from the ground just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

(a) 45° (b) $\tan^{-1}\left(\frac{bc}{a(c-a)}\right)$

(c) $\tan^{-1}\frac{bc}{a^2}$ (d) $\tan^{-1}\frac{b^2}{ca}$

7. $\lim\limits_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots+\sqrt{n}}{4n+5n^{3/2}-3\sqrt{n^3+10n^2+5n+6}} =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

8. Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n-sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, then $n =$

(a) 5 (b) 6 (c) 7 (d) 8

9. The value of $\cot\left\{\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right\}$ is

(a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$ (d) $\frac{24}{23}$

10. If the positive numbers a, b, c are the p^{th}, q^{th}, r^{th} terms of a G.P., then the vectors
$\hat{i}\log a + \hat{j}\log b + \hat{k}\log c$
and $(q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$ are
(a) equal (b) parallel
(c) perpendicular (d) None of these

11. The coordinate axes are rotated about the origin O in the counter-clockwise direction through an angle 60°. If p and q are the intercepts made on the new axes by a straight line whose equation referred to the original axes is $x + y = 1$, then $\frac{1}{p^2}+\frac{1}{q^2} =$
(a) 2 (b) 4 (c) 6 (d) 8

12. If $f : [-2a, 2a] \to R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$. If the left hand derivative of $f(x)$ at $x = a$ is zero, then the left hand derivative of $f(x)$ at $x = -a$ is
(a) 1 (b) – 1
(c) 0 (d) none of these

13. Let $f(x)$ be an even function,
$$I_1 = \int_0^{\pi/2} f(\cos 2x)\cos x\,dx,\ I_2 = \int_0^{\pi/4} f(\sin 2x)\cos x\,dx$$
Then $\frac{I_1}{I_2} =$
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

14. If α, β are the roots of $\lambda(x^2 + x) + x + 5 = 0$ and λ_1, λ_2 are the two values of λ for which α, β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=4$, then the value of $\frac{\lambda_1}{\lambda_2}+\frac{\lambda_2}{\lambda_1} =$
(a) 254 (b) 482 (c) 784 (d) 782

15. ABC is a triangle in a plane with vertices $A(2, 3, 5)$, $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is
(a) 1130 (b) 1348
(c) 1077 (d) 676

16. Let $a=\lim_{x\to 1}\left(\frac{x}{\ln x}-\frac{1}{x\ln x}\right); b=\lim_{x\to 0}\frac{x^3-16x}{4x+x^2};$
$c=\lim_{x\to 0}\frac{\ln(1+\sin x)}{x}$ and
$d=\lim_{x\to -1}\frac{(x+1)^3}{3\left(\sin(x+1)-(x+1)\right)}$,
then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is
(a) Singular (b) Null
(c) Identity (d) None of these

17. Let the sequence $a_1, a_2, a_3, ..., a_n, ...$ form an A.P. Then, $a_1^2 - a_2^2 + a_3^2 - a_4^2 + ... + a_{2n-1}^2 - a_{2n}^2$ is equal to
(a) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (b) $\frac{2n}{2n-1}(a_{2n}^2 - a_1^2)$
(c) $\frac{n}{n+1}(a_1^2 - a_{2n}^2)$ (d) None of these

18. If the area bounded by the parabola $y^2 = 4ax$ and the double ordinate $x = 6$ is twice the area bounded by the parabola and its latus rectum, then the length of the latus rectum is
(a) $6\sqrt{2}$ (b) $12\sqrt{2}$ (c) $6\cdot 3^{1/3}$ (d) $12\cdot 2^{1/3}$

19. Let $f : (0, \infty) \to R$ be a differentiable function such that $f'(x)=2-\frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then
(a) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right)=1$ (b) $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right)=2$
(c) $\lim_{x\to 0^+} x^2 f'(x)=0$
(d) $|f(x)| \le 2$ for all $x \in (0, 2)$

20. The value of $\lim_{n\to\infty}\prod_{r=3}^{n}\frac{r^3+2^3}{r^3-2^3}$ is equal to
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

SECTION-B (NUMERICAL VALUE TYPE)

21. The probability that the roots of the equation $x^2 + nx + \frac{n}{2} = 0$ are real, $n \in N$ and $n \le 5$, is $\frac{m}{n}$, then $m =$ _____.

22. If $[x]^2 + [x - 2] < 0$ and $\{x\} = \frac{1}{2}$, then the number of possible values of x, is ($[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x, respectively) _____.

23. The last digit of the constant term in the expansion of $\left(1+x+\frac{1}{x^2}+\frac{1}{x^3}\right)^6$ is _____.

24. Let $0 < a < b < \frac{\pi}{2}$. If $f(x) = \begin{vmatrix} \tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \end{vmatrix}$, then minimum possible number of roots of $f'(x) = 0$ in (a, b) is _____.

25. Let A be the set of all 3×3 symmetric matrices all of whose entries either 0 or 1. Five of these entries are 1 each and four of them are 0 each. If n is the number of such matrices, then $n/2$ is _____.

SOLUTIONS

1. (c) : The total length of the interval = 12 – (– 6) = 18.
If graph of $y = -x^2 + 2(a + 4)x - (3a + 40)$ is entirely below x-axis, the value of discriminant of the above quadratic expression must be negative.

$\therefore 4(a+4)^2 - 4(1)(3a+40) < 0$

$\Rightarrow a^2 + 5a - 24 < 0 \Rightarrow (a+8)(a-3) < 0 \Rightarrow -8 < a < 3$

but $a \in [-6, 12]$

$\therefore -6 \le a < 3$ for event to happen

$\therefore$ Length of interval = 3 – (– 6) = 9

Hence, the required probability $= \frac{9}{18} = \frac{1}{2}$

2. (b) : We have, $f(x) = \log_{e^2 x}\left(\frac{2\ln x + 2}{-x}\right)$ and $g(x) = \{x\}$

For $f(x)$ to be real, we must have

$x > 0, x \ne \frac{1}{e^2}$ and $\left(\frac{2\ln(x)+2}{-x}\right) > 0$

Now, $\frac{(2\ln x + 2)}{-x} > 0$ and $x > 0$

$\Rightarrow 2\ln x + 2 < 0 \Rightarrow \ln x < -1 \Rightarrow x < e^{-1} \Rightarrow x < \frac{1}{e}$

Thus, $f(x)$ is defined if $x \in \left(0, \frac{1}{e}\right) - \left\{\frac{1}{e^2}\right\}$.

Hence, range of $g(x)$ should be $\left(0, \frac{1}{e}\right) - \left\{\frac{1}{e^2}\right\}$.

3. (c) : Using the principle of inclusion and exclusion, we have the number of ways in which card number 1 be placed in envelope number 2

$= 5! - \{{}^4C_1 \cdot 4! - {}^4C_2 \cdot 3! + {}^4C_3 \cdot 2! - {}^4C_4 \cdot 1!\}$

$= 120 - \{96 - 36 + 8 - 1\} = 120 - \{60 + 8 - 1\} = 53$

4. (c) : $g'(x) = f'\left(\frac{x}{2}\right) - f'(2-x) > 0$ if $f'\left(\frac{x}{2}\right) > f'(2-x)$

$\Rightarrow \frac{x}{2} < 2 - x$

[Because $f'(x)$ is a decreasing function $(f''(x) < 0)$]

$\therefore x < 4 - 2x \Rightarrow x < \frac{4}{3}$. If $x \in \left(0, \frac{4}{3}\right)$, $g(x)$ increases and decreases in $\left(\frac{4}{3}, 2\right)$.

5. (c) : If A is singular there exists a non-zero vector X such that $AX = 0$ *i.e.*,

$a_1x + a_2y + a_3z = 0$ ...(i)

$b_1x + b_2y + b_3z = 0$ ...(ii)

$c_1x + c_2y + c_3z = 0$ ...(iii)

Let $|x|$ be the max $\{|x|, |y|, |z|\}$

(i) $\Rightarrow -a_1 = a_2\frac{y}{x} + a_3\frac{z}{x}$

$|-a_1| = \left|a_2\frac{y}{x} + a_3\frac{z}{x}\right| \le |a_2|\left|\frac{y}{x}\right| + |a_3|\left|\frac{z}{x}\right|$

$\Rightarrow |a_1| \le |a_2| + |a_3|$, a contradiction.

Hence, A is non-singular.

6. (b) : The parabola has its axis vertical.

Let it be $y = px^2 + qx + r$

The parabola passes through (0, 0)

$\therefore r = 0$

The parabola passes through $(c, 0)$

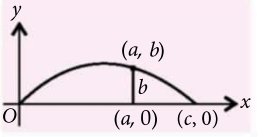
$\therefore pc^2 + qc = 0 \Rightarrow q = -pc$

The parabola is $y = p(x^2 - cx)$

It passes through (a, b)

$\therefore b = p(a^2 - ca) = -ap(c - a)$

$\Rightarrow p = \frac{-b}{a(c-a)}$



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The National Testing Agency is conducting the **Joint Entrance Examination (Main)–2025 Session–2** at different Centres located in various **cities** throughout the country and **15 cities outside India**. The schedule of examination for this session will be as follows :

Date of Exam	Paper	Shift
02, 03, 04, 07 April 2025	Paper 1 (B.E./B. Tech)	First Shift (09:00 A.M. to 12:00 Noon) and Second Shift (03:00 P.M. to 6:00 P.M.)
08 April 2025	Paper 1 (B.E./B. Tech)	Second Shift (03:00 P.M. to 6:00 P.M.)
09 April 2025	Paper 2A (B. Arch), Paper 2B (B. Planning) and Paper 2A & 2B (B. Arch & B. Planning both)	First Shift (09:00 A.M. to 12:30 P.M.)

The Candidates are advised to visit the official websites of NTA (**www.nta.ac.in**) and (**https://jeemain.nta.nic.in/**) for the latest updates.

The parabola is $y = \frac{b}{a(c-a)}(cx - x^2)$

$\frac{dy}{dx} = \frac{b(c-2x)}{a(c-a)}$

The angle of projection is

$\tan^{-1}\left(\frac{dy}{dx}\right)_{\text{at }(0,0)} = \tan^{-1}\left(\frac{bc}{a(c-a)}\right)$

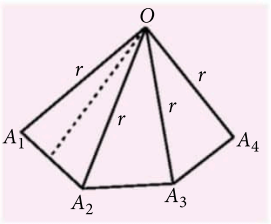
7. (c) : $L = \lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{n^{3/2}} \cdot \frac{1}{\frac{4}{\sqrt{n}}+5-3\sqrt{1+\frac{10}{n}}+...}$

$= \frac{1}{2}\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\sqrt{\frac{r}{n}} = \frac{1}{2}\int_0^1 \sqrt{x}\,dx = \frac{1}{3}.$

8. (c) : Let $\angle A_1OA_2 = \theta$, $OA_1 = r$

$A_1A_2 = 2r\sin\frac{\theta}{2}$, $A_1A_3 = 2r\sin\theta$,

$A_1A_4 = 2r\sin\frac{3\theta}{2}$



$\therefore \quad \frac{1}{\sin\theta}+\frac{1}{\sin\frac{3\theta}{2}} = \frac{1}{\sin\frac{\theta}{2}}$

$\Rightarrow \quad \sin\frac{3\theta}{2}+\sin\theta = \frac{\sin\theta\sin\frac{3\theta}{2}}{\sin\frac{\theta}{2}} = 2\cos\frac{\theta}{2}\sin\frac{3\theta}{2}$

$= \sin 2\theta + \sin\theta$

$\Rightarrow \quad \sin\frac{3\theta}{2} = \sin 2\theta = \sin(\pi - 2\theta)$

$\Rightarrow \quad \frac{3\theta}{2} = \pi - 2\theta \Rightarrow \theta = \frac{2\pi}{7}$

But $\theta = \frac{2\pi}{n} \quad \therefore \quad n = 7$

9. (b) : $\cot\sum_{n=1}^{23}\left(\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$

$= \cot\sum_{n=1}^{23}\cot^{-1}\left(1+\frac{2(n+1)n}{2}\right) = \cot\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))$

$= \cot\sum_{n=1}^{23}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \cot(\tan^{-1}24 - \tan^{-1}1)$

$= \cot\left(\tan^{-1}\left(\frac{24-1}{1+24}\right)\right) = \cot\left(\tan^{-1}\frac{23}{25}\right)$

$= \cot\left(\cot^{-1}\frac{25}{23}\right) = \frac{25}{23}$

10. (c) : Let the G.P. be $\alpha, \alpha\beta, \alpha\beta^2$... and

$a = \alpha\beta^{p-1}$, $b = \alpha\beta^{q-1}$, $c = \alpha\beta^{r-1}$

$\log a = \log\alpha + (p-1)\log\beta$, $\log b = \log\alpha + (q-1)\log\beta$,

$\log c = \log\alpha + (r-1)\log\beta$

Let $\vec{u} = \hat{i}\log a + \hat{j}\log b + \hat{k}\log c$

$= \hat{i}(\log\alpha + (p-1)\log\beta) + \hat{j}(\log\alpha + (q-1)\log\beta)$

$+ \hat{k}(\log\alpha + (r-1)\log\beta)$

and $\vec{v} = (q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$

$\therefore \quad \vec{u}\cdot\vec{v} = \Sigma(\log\alpha + (p-1)\log\beta)(q-r)$

$= (\log\alpha - \log\beta)\Sigma(q-r) + \log\beta\,\Sigma\,p(q-r) = 0 + 0 = 0.$

11. (a) : If the axes are rotated about the origin in anticlockwise direction, the relations relating old coordinates (x, y) and new coordinates (X, Y) are

$x = X\cos 60° - Y\sin 60°$ and $y = X\sin 60° + Y\cos 60°$

$\Rightarrow \quad x = \frac{X-\sqrt{3}Y}{2}$ and $y = \frac{\sqrt{3}X+Y}{2}$

The equation of the line $x + y = 1$ with reference to new coordinate axes is $\frac{X-\sqrt{3}Y}{2}+\frac{\sqrt{3}X+Y}{2} = 1$

$\Rightarrow \quad (\sqrt{3}+1)X + (1-\sqrt{3})Y = 2$

This intercepts lengths p and q on the new coordinate axes.

$\therefore \quad p = \frac{2}{\sqrt{3}+1}$ and $q = \frac{2}{1-\sqrt{3}}$

$\Rightarrow \quad \frac{1}{p^2}+\frac{1}{q^2} = \frac{(\sqrt{3}+1)^2+(1-\sqrt{3})^2}{4} = 2$

12. (c) : $0 = f'(a) = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$

$= \lim_{h\to 0}\frac{f(a+h)-f(a)}{-h}$ (since $f(x) = f(2a-x)$)

$= \lim_{h\to 0}\frac{-f(-a-h)+f(-a)}{-h}$ (since $f(x)$ is odd)

$= -\lim_{h\to 0}\frac{f(-a-h)-f(-a)}{-h} = -f'(-a^-)$

$\therefore \quad f'(-a^-) = 0$

13. (d) : $I_1 = \int_0^{\pi/4} f(\cos 2x)\cos x\,dx + \int_{\pi/4}^{\pi/2} f(\cos 2x)\cos x\,dx$

Setting $x = \frac{\pi}{4} - t$ in the first integral and $x = \frac{\pi}{4} + t$ in the second integral,

$$I_1 = \int_0^{\pi/4} f(\sin 2t)\cos\left(\frac{\pi}{4}-t\right)dt + \int_0^{\pi/4} f(\sin 2t)\cos\left(\frac{\pi}{4}+t\right)dt$$

$$= \int_0^{\pi/4} f(\sin 2t)\left[\cos\left(\frac{\pi}{4}-t\right)+\cos\left(\frac{\pi}{4}+t\right)\right]dt$$

$$= \sqrt{2}\int_0^{\pi/4} f(\sin 2t)\cos t\,dt = \sqrt{2}I_2 \Rightarrow \frac{I_1}{I_2} = \sqrt{2}$$

14. (d) : α and β are the roots of $\lambda x^2 + (\lambda + 1)x + 5 = 0$,

$\therefore \quad \alpha+\beta = -\left(\frac{\lambda+1}{\lambda}\right), \alpha\beta = \frac{5}{\lambda}; \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = 4$

$\Rightarrow \alpha^2 + \beta^2 = 4\alpha\beta, (\alpha+\beta)^2 = 6\alpha\beta$

$\Rightarrow (\lambda + 1)^2 = 30\lambda$

$\Rightarrow \lambda^2 - 28\lambda + 1 = 0$

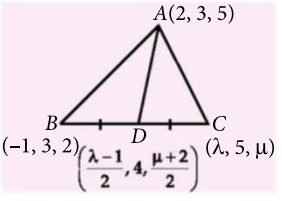
$\Rightarrow \lambda_1 + \lambda_2 = 28, \lambda_1\lambda_2 = 1$

$$\frac{\lambda_1}{\lambda_2}+\frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2+\lambda_2^2}{\lambda_1\lambda_2} = (\lambda_1+\lambda_2)^2 - 2 = 28^2 - 2 = 782$$

15. (b) : *D.r's* of AD are $\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,



$$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$$

$\Rightarrow \lambda = 7, \mu = 10$

$\therefore \quad \lambda^3 + \mu^3 + 5 = 1348$

16. (a) : $a = 2;\ b = -4;\ c = 1;\ d = -2$

$$\therefore\ A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

$|A| = -4 + 4 = 0 \Rightarrow A$ is singular

17. (a) : Let d be the common difference of the A.P. Then,

$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_{2n} - a_{2n-1}$.

Now, $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2$

$= (a_1 + a_2)(a_1 - a_2) + (a_3 + a_4)(a_3 - a_4) + \ldots$

$\quad + (a_{2n-1} + a_{2n})(a_{2n-1} - a_{2n})$

$= -d(a_1 + a_2 + a_3 + \ldots + a_{2n})$

$$= -dn(a_1 + a_{2n}) = -dn\frac{(a_1^2 - a_{2n}^2)}{a_1 - a_{2n}}$$

$$= \frac{dn(a_1^2 - a_{2n}^2)}{a_{2n} - a_1} = \frac{dn(a_1^2 - a_{2n}^2)}{(2n-1)d} = \frac{n(a_1^2 - a_{2n}^2)}{(2n-1)}$$

18. (d) : We have, $2\int_0^6 y\,dx = 2\times 2\int_0^a y\,dx$

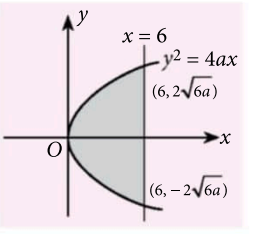
$\Rightarrow \int_0^6 \sqrt{x}\,dx = 2\int_0^a \sqrt{x}\,dx$

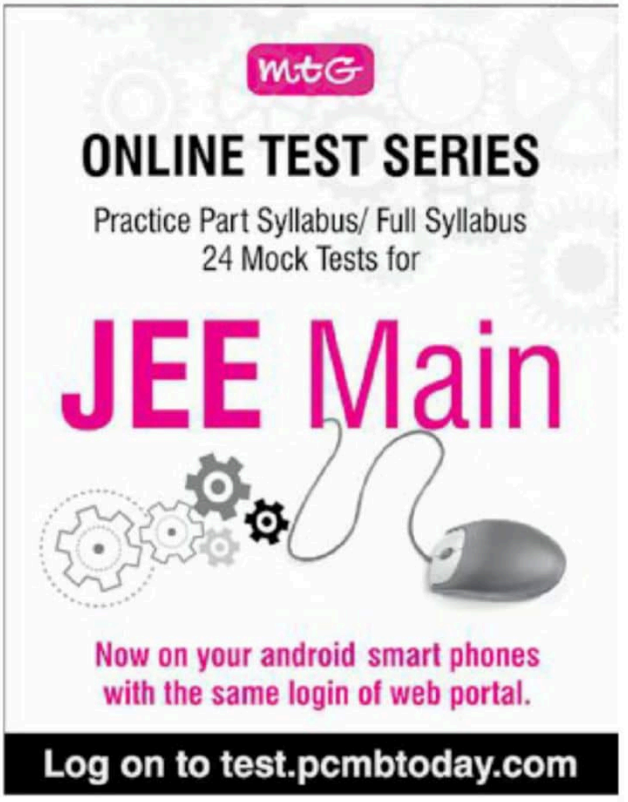
$\Rightarrow \frac{2}{3}\times 6\sqrt{6} = 2\times\frac{2}{3}a\sqrt{a}$

$\Rightarrow a^{3/2} = 3\sqrt{6}$

$\Rightarrow a = 3\left(2^{1/3}\right) = 3\left(2^{1/3}\right)$

$\therefore \quad 4a = 12(2)^{1/3}$





19. (a) : $f'(x)=2-\dfrac{f(x)}{x}$

$\Rightarrow \quad f'(x)+\dfrac{1}{x}f(x)=2$ is linear differential equation.

Hence, I.F. $=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$

Thus the solution is given by $x\cdot f(x)=\int 2x\,dx+\lambda$

i.e., $xf(x)=x^2+\lambda$. As $f(1)\neq 1$, we have $\lambda\neq 0$

$\therefore \quad f(x)=x+\dfrac{\lambda}{x},\ \lambda\neq 0$

Thus, $f'(x)=1-\dfrac{\lambda}{x^2},\ \lambda\neq 0$

Now, $\lim\limits_{x\to 0^+} f'\left(\dfrac{1}{x}\right)=\lim\limits_{x\to 0^+}(1-\lambda x^2)=1$

$\lim\limits_{x\to 0^+} x\,f\left(\dfrac{1}{x}\right)=\lim\limits_{x\to 0^+} x\left(\dfrac{1}{x}+\lambda x\right)=\lim\limits_{x\to 0^+}(1+\lambda x^2)=1$

$\lim\limits_{x\to 0^+} x^2 f'(x)=\lim\limits_{x\to 0^+} x^2\left(1-\dfrac{\lambda}{x^2}\right)=\lim\limits_{x\to 0^+}(x^2-\lambda)=-\lambda$

Again, $\lim\limits_{x\to 0^+} f(x)\to\infty$

Hence the function is not bounded.

Note that λ can be –ve or +ve.

20. (c) : $\lim\limits_{n\to\infty}\prod\limits_{r=3}^{n}\dfrac{(r+2)(r^2-2r+2^2)}{(r-2)(r^2+2r+2^2)}$

$=\lim\limits_{n\to\infty}\prod\limits_{r=3}^{n}\dfrac{r+2}{r-2}\cdot\prod\limits_{r=3}^{n}\dfrac{r^2-2r+2^2}{r^2+2r+2^2}$

$=\lim\limits_{n\to\infty}\dfrac{5}{1}\cdot\dfrac{6}{2}\cdot\dfrac{7}{3}\cdot\dfrac{8}{4}\cdot\dfrac{9}{5}\cdots\dfrac{n+2}{n-2}\times\dfrac{7}{19}\cdot\dfrac{12}{28}\cdot\dfrac{19}{39}\cdots\dfrac{n^2-2n+4}{n^2+2n+4}$

$=\lim\limits_{n\to\infty}\dfrac{(n-1)n(n+1)(n+2)}{1\cdot 2\cdot 3\cdot 4}\cdot\dfrac{7\times 12}{(n^2+3)(n^2+2n+4)}$

$=\lim\limits_{n\to\infty}\dfrac{7}{2}\dfrac{\left(1-\dfrac{1}{n}\right)\left(1+\dfrac{1}{n}\right)\left(1+\dfrac{2}{n}\right)}{\left(1+\dfrac{3}{n^2}\right)\left(1+\dfrac{2}{n}+\dfrac{4}{n^2}\right)}=\dfrac{7}{2}$

21. (4) : Roots are real if $D\geq 0$ *i.e.*, if $n^2-4\left(\dfrac{n}{2}\right)\geq 0$

i.e. if $n^2-2n\geq 0$ ...(i)

We find that n = 2, 3, 4 and 5 satisfy the inequality (i). So four out of 5 choices are favourable. Hence, required probability = $\dfrac{4}{5}$

22. (2) : $[x]^2+[x-2]<0$

$\Rightarrow \quad [x]^2+[x]-2<0 \Rightarrow ([x]+2)([x]-1)<0$

$\Rightarrow \quad -2<[x]<1\Rightarrow x\in[-1,1)$

As, $\{x\}=\dfrac{1}{2}\Rightarrow x=\dfrac{-1}{2},\dfrac{1}{2}$

23. (6) : $\left(1+x+\dfrac{1}{x^2}+\dfrac{1}{x^3}\right)^6=\left((1+x)\dfrac{(1+x^3)}{x^3}\right)^6$

$\therefore$ The constant term required is the coefficient of x^{18} in the expansion of $(1+x)^6(1+x^3)^6$

$=\left(1+\ldots\binom{6}{3}x^3+\ldots+x^6\right)$

$\left(1+\binom{6}{1}x^3+\binom{6}{2}x^6+\ldots+\binom{6}{6}x^{18}\right)$

$=1\cdot\binom{6}{6}+\binom{6}{3}\binom{6}{5}+1\cdot\binom{6}{4}=1+120+15=136$

24. (1) : $f(x)=\begin{vmatrix}\tan x & \tan a & \tan b\\ \sin x & \sin a & \sin b\\ \cos x & \cos a & \cos b\end{vmatrix}$

$f(a)=f(b)=0$, $f(x)$ is continuous in $[a,b]$ and differentiable in (a,b).

Hence, $f'(x)=0$ has at least one root in (a,b).

25. (6) : We have five entries 1 and the remaining four entries are 0.

Since the matrix is symmetric, we must have an even number of zeros for $i\neq j$.

Hence, we have two cases:

(i) Two entries in the diagonal are 0.

We can select 2 places from three (in diagonal) in ${}^3C_2$ ways. Now, we have to select elements for the upper triangle.

For the upper triangle, we have three places of which one entry is 0 and two entries are 1. One place from 3 can be selected in ${}^3C_1$ ways.

Hence, number of matrices is ${}^3C_2\times{}^3C_1=9$.

(ii) If all the entries in the principal diagonal is 1, we have two 0 and one 1 in the upper triangle. Hence, number of matrices is 3.

Thus, total matrices = 12.

◈◈

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(d)
6.	(c)	**7.**	(b,c,d)	**8.**	(b,d)	**9.**	(c,d)	**10.**	(a,d)
11.	(b,c)	**12.**	(a,b,c,d)	**13.**	(a,b,c)	**14.**	(b)	**15.**	(d)
16.	(a)	**17.**	(23)	**18.**	(4)	**19.**	(1)	**20.**	(8)

JEE 2025 ADVANCED

PRACTICE PAPER

Exam on 18th May

PAPER-I

SECTION 1 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

1. Let $f(x) = |x - 1|$. Then

(a) $f(x^2) = (f(x))^2$ (b) $f(x + y) = f(x) + f(y)$

(c) $f(|x|) = |f(x)|$ (d) none of these

2. Let $f(x)=\dfrac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and

$g(x)=\underbrace{(f\ o\ f\ o....o\ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then

$\int x^{n-2}\, g(x)\, dx$ equals

(a) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

(b) $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(c) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

(d) $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

3. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has component $p + 1$ and 1, then

(a) $p \neq 0$ (b) $p = 1$ or $p=\dfrac{-1}{3}$

(c) $p = -1$ or $p=\dfrac{1}{3}$ (d) $p = 1$ or $p = -1$

4. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2-2C_1^2+.....+(-1)^n(n+1)C_n^2\right]$$

where n is an even positive integer, is equal to

(a) 0 (b) $(-1)^{n/2}(n + 1)$

(c) $(-1)^n(n + 2)$ (d) none of these

SECTION 2 (MAXIMUM MARKS : 12)

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –2 In all other cases.

5. For a positive interger n, let

$f_n(\theta)=(\tan\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)$

..... $(1+\sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_5\left(\frac{\pi}{128}\right)=1$

6. For every integer n, let a_n and b_n be real numbers. Let function $f: R \to R$ be given by

$$f(x)=\begin{cases} a_n+\sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n+\cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}$$

for all integers n. If f is continuous, then which of the following hold(s) for all n?

(a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$

(c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

7. Let $f: R \to R, g: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then

(a) $g'(2)=\frac{1}{15}$ (b) $h'(1) = 666$

(c) $h(0) = 16$ (d) $h(g(3)) = 36$

SECTION 3 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Let $\omega = e^{i\pi/3}$ and a, b, c, x, y, z be non-zero complex numbers such that $a + b + c = x$, $a + b\omega + c\omega^2 = y$, $a + b\omega^2 + c\omega = z$. Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is _____.

9. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x \in R$, where $f'(x)$ denotes $\frac{d f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then, the value of $y(2)$ is _____ .

10. Let $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right), -\frac{\pi}{4}<\theta<\frac{\pi}{4}$.

Then, the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is _____.

11. One of the diameter of the circle circumscribing the rectangle $ABCD$, $4y = x + 7$. If A and B are the points $(-3, 4)$, and $(5, 4)$ respectively, then the area (in sq. units) of the rectangle is _____.

12. Let k be a positive real number and let

$$A=\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$B=\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If det (adj A) + det(adj B) = 10^6, then $[k]$ is equal to _____.

[**Note :** adj M denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k].

13. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals _____.

SECTION 4 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) Matching List Sets.*
- *Each set has ONE Multiple Choice Question.*
- *Each set has TWO lists: List-I and List-II.*
- *List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).*
- *FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

14. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are NOT in the committee together.

List-I		List-II	
(P)	The value of α_1 is	(1)	136
(Q)	The value of α_2 is	(2)	189
(R)	The value of α_3 is	(3)	461
(S)	The value of α_4 is	(4)	200
		(5)	381

The correct option is :

(a) (P) → (4), (Q) → (3), (R) → (2), (S) → (1)

(b) (P) → (1), (Q) → (4), (R) → (2), (S) → (3)

(c) (P) → (4), (Q) → (3), (R) → (5), (S) → (2)

(d) (P) → (4), (Q) → (2), (R) → (3), (S) → (1)

15. A line $L : y = mx + 3$ meets y-axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List-I with List-II and select the correct answer using the code given below the lists:

List-I		List-II	
(P)	$m =$	(1)	1/2
(Q)	Maximum area of ΔEFG is	(2)	4
(R)	$y_0 =$	(3)	2
(S)	$y_1 =$	(4)	1
		(5)	3

The correct option is :

(a) (P) → (4), (Q) → (1), (R) → (2), (S) → (3)

(b) (P) → (3), (Q) → (4), (R) → (1), (S) → (5)

(c) (P) → (5), (Q) → (3), (R) → (2), (S) → (4)

(d) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)

16. Match the statements given in List-I with the values given in List-II .

List-I		List-II	
(P)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(1)	$\pi/6$
(Q)	If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(2)	$2\pi/3$
(R)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(3)	$\pi/3$
(S)	The maximum value of $\left\lvert \text{Arg}\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1$, $z \ne 1$ is given by	(4)	π
		(5)	$\pi/2$

Maximum arg ω approaches π/2 but doesn't attain.

The correct option is :

(a) (P) → (2), (Q) → (1), (R) → (4), (S) → (5)

(b) (P) → (3), (Q) → (2), (R) → (4), (S) → (1)

(c) (P) → (4), (Q) → (2), (R) → (3), (S) → (1)

(d) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)

17. Match the sets given in List-I with their number of elements given in List-II.

List-I		List-II	
(P)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(1)	has two elements
(Q)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(2)	has three elements
(R)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(3)	has four elements
(S)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(4)	has five elements
		(5)	has six elements

The correct option is :

(a) (P) → (1), (Q) → (4), (R) → (1), (S) → (4)

(b) (P) → (1), (Q) → (1), (R) → (5), (S) → (3)

(c) (P) → (2), (Q) → (1), (R) → (5), (S) → (4)

(d) (P) → (2), (Q) → (4), (R) → (1), (S) → (3)

SECTION 1 (MAXIMUM MARKS: 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

1. Let a, b, c be non zero real numbers such that

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\,dx$$

Then the quadratic equation $ax^2 + bx + c = 0$ has
(a) no root in (0, 2)
(b) at least one root in (1, 2)
(c) a double root in (0, 2)
(d) two imaginary roots

2. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$ then the value of $\lim\limits_{x\to a} \dfrac{g(x)f(a)-g(a)f(x)}{x-a}$ is

(a) –5 (b) $\dfrac{1}{5}$
(c) 5 (d) none of these

3. The area of the region between the curves $y=\sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y=\sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x=\dfrac{\pi}{4}$ is

(a) $\displaystyle\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (b) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(c) $\displaystyle\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (d) $\displaystyle\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

4. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a) $\dfrac{5}{2}$ (b) $\dfrac{11}{2}$ (c) 6 (d) $\dfrac{13}{2}$

SECTION 2 (MAXIMUM MARKS : 12)

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : –2 In all other cases.

5. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

(a) $a > 0, b > 0$ (b) $a > 0, b < 0$
(c) $a < 0, b > 0$ (d) $a < 0, b < 0$

6. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1,$$

$$\tan(2\pi-\theta) > 0 \text{ and } -1 < \sin\theta < -\frac{\sqrt{3}}{2}.$$

Then φ cannot satisfy

(a) $0 < \varphi < \dfrac{\pi}{2}$ (b) $\dfrac{\pi}{2} < \varphi < \dfrac{4\pi}{3}$

(c) $\dfrac{4\pi}{3} < \varphi < \dfrac{3\pi}{2}$ (d) $\dfrac{3\pi}{2} < \varphi < 2\pi$

7. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' $\left(\text{here } y' = \dfrac{dy}{dx}, y'' = \dfrac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true?

(a) $P = y + x$
(b) $P = y - x$
(c) $P + Q = 1 - x + y + y' + (y')^2$
(d) $P - Q = x + y - y' - (y')^2$

SECTION 3 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

8. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is _____.

9. Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is _____.

10. The value of

$$6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}}}}\ldots.\right)$$

is _____.

11. For each positive integer n, let

$$y_n = \frac{1}{n}((n+1)(n+2)\ldots(n+n))^{\frac{1}{n}}.$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim\limits_{n\to\infty} y_n = L$, then the value of $[L]$ is _____

12. Let S be the sample space of all 3×3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S :$ sum of entries of A is $7\}$.

If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals _____.

13. The sum $\sum\limits_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, is maximum when m is _____.

SECTION 4 (MAXIMUM MARKS : 12)

- *This section contains TWO (02) paragraphs.*
- *Based on each paragraph, there are TWO (02) questions.*
- *The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

PARAGRAPH-I

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i =1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

14. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and NONE of the remaining students gets the seat previously allotted to him/her, is $\frac{m}{n}$, find $m + n$ = _____.

15. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is $\frac{7}{p}$, then p is _____.

PARAGRAPH-II

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

16. If the equation of a common tangent with positive slope to the circle as well as to the hyperbola is $ax - \sqrt{5}y + b = 0$ then $\frac{b}{a}$ is _____.

17. If the equation of the circle with AB as its diameter is $ax^2 + by^2 - cx + d = 0$ then $a + b + c + d$ is _____.

SOLUTIONS

PAPER-I

1. (d) : $f(x) = |x-1| = \begin{cases} x-1 & x \geq 1 \\ 1-x & x < 1 \end{cases}$

Consider, $f(x^2) = (f(x))^2$

If it is true, it should be true for all x

$\therefore$ Put $x = 2 \Rightarrow f(2^2) = f(4) \neq (f(2))^2$

$\therefore$ (a) is not correct

Consider $f(x + y) = f(x) + f(y)$

Put $x = 2, y = 3$.

$\Rightarrow f(5) \neq f(2) + f(3)$ $\quad\therefore$ (b) is not correct

Consider $f(|x|) = |f(x)|$

Put $x = -4$, $f(|-4|) = f(4) = 3$

$|f(-4)| = |1 - (-4)| = 5$

$\therefore$ (c) is not correct.

2. (a) : Given that $f(x) = \dfrac{x}{(1+x^n)^{\frac{1}{n}}}$

$$\therefore \quad f(f(x)) = \frac{y}{(1+y^n)^{\frac{1}{n}}}$$

$$= \frac{x}{(1+2x^n)^{\frac{1}{n}}} \qquad \left[\because y = \frac{x}{(1+x^n)^{\frac{1}{n}}}\right]$$

Similarly, $f(f(f(x))) = \dfrac{x}{(1+3x^n)^{\frac{1}{n}}}$

$$\Rightarrow \underbrace{(f\,o\,f\,o\,f\,o\,....\,f)}_{n \text{ times}}(x)\frac{x}{(1+nx^n)^{1/n}} = g(x)$$

$$\int x^{n-2}\, g(x)\, dx = \int \frac{x^{n-1}}{(1+nx^n)^{\frac{1}{n}}}\, dx$$

$$= \frac{1}{n^2}\int n^2 \cdot x^{n-1} \cdot (1+nx^n)^{-\frac{1}{n}}\, dx$$

$$= \frac{1}{n^2}\cdot\frac{(1+nx^n)^{1-\frac{1}{n}}}{1-\frac{1}{n}} + K = \frac{\left(1+nx^n\right)^{1-\frac{1}{n}}}{n(n-1)} + K$$

3. (b) : Let $\vec{a}$ makes angle α with original x-axis.

Let the original co-ordinate system be rotated through angle θ, we have

$2p = a\cos\alpha$, $1 = a\sin\alpha$ $\qquad [\text{Taking } |\vec{a}| = a]$

$p + 1 = a\cos(\alpha + \theta)$, $1 = a\sin(\alpha + \theta)$

$$\therefore \quad \begin{cases} p+1 = a\cos\theta\cdot\cos\alpha - a\sin\theta\cdot\sin\alpha \\ 1 = a\sin\alpha\cos\theta + a\cos\alpha\cdot\sin\theta \end{cases}$$

$$\therefore \quad \begin{cases} p+1 = 2p\cos\theta - \sin\theta & ....(1) \\ 1 = \cos\theta + 2p\sin\theta & ....(2) \end{cases}$$

Squaring (1) and (2) and adding, we get

$p^2 + 2p + 2 = 4p^2 + 1$

$\Rightarrow 3p^2 - 2p - 1 = 0 \quad \Rightarrow \quad p = 1$ or $-\dfrac{1}{3}$.

4. (d) : When n is an even positive integer, n is of the form $2K$, $K \in N$

Let $f(K) = \dfrac{2\lfloor K \rfloor K}{\lfloor 2K}$

$$\left[C_0^{\,2} - 2.C_1^{\,2} + 3C_2^{\,2} + ... + (-1)^{2K}(2K+1)C_{2K}^2\right]$$

Reversing the order of terms,

$$f(K) = \frac{2\lfloor K \rfloor K}{\lfloor 2K}\left[(2K+1)C_0^{\,2} - 2K.C_1^{\,2} + ...+C_{2K}^2\right]$$

$$\Rightarrow 2f(K) = \frac{2\lfloor K \rfloor K}{\lfloor 2K}(2K+2)(C_0^{\,2} - C_1^{\,2} + C_2^{\,2} ... C_{2K}^2)$$

$$(\because\ C_0^{\,2} - C_1^{\,2} + C_2^{\,2} - ... + C^2{}_{2K} = (-1)^{K\ 2K}C_K)$$

$$\Rightarrow \quad 2f(K) = \frac{2}{{}^{2K}C_K}2(K+1)(-1)^{K\ 2K}C_K$$

$$\therefore \quad f(K) = (-1)^{n/2}\left(\frac{n}{2}+1\right)\cdot 2 = (-1)^{n/2}(n+2)$$

5. (a, b, c, d) :

We have, $\left(\tan\dfrac{\theta}{2}\right)(1+\sec\theta) = \dfrac{\sin\theta/2}{\cos\theta/2}\left(1+\dfrac{1}{\cos\theta}\right)$

$$= \frac{\sin(\theta/2)\cdot 2\cos^2\theta/2}{\cos(\theta/2)\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Thus, $f_n(\theta)$

$= \left(\tan\dfrac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\,.....\,(1+\sec 2^n\theta)$

$= (\tan\theta)\,(1+\sec 2\theta)\,(1+\sec 4\theta).....\,(1+\sec 2^n\theta)$

$= (\tan 2\theta)\,(1+\sec 4\theta)\,.....\,(1+\sec 2^n\theta)$

$= (\tan 4\theta)\,(1+\sec 8\theta)\,.....\,(1+\sec 2^n\theta)$

$= (\tan 8\theta)(1+\sec 16\theta)\,.....\,(1+\sec 2^n\theta) = \tan(2^n\theta)$

Now, $f_2\left(\dfrac{\pi}{16}\right) = \tan\left(2^2.\dfrac{\pi}{16}\right) = \tan\left(\dfrac{\pi}{4}\right) = 1$

$$f_3\left(\frac{\pi}{32}\right) = \tan\left(2^3\frac{\pi}{32}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$f_4\left(\frac{\pi}{64}\right) = \tan\left(2^4\frac{\pi}{64}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

and $f_5\left(\dfrac{\pi}{128}\right) = \tan\left(2^5\dfrac{\pi}{128}\right) = \tan\left(\dfrac{\pi}{4}\right) = 1$

6. (b, d) : At the points $x = 2n$, we have

$f(2n) = a_n + \sin 2n\pi = a_n$

Also, for the L.H.L., we have

$$\text{L.H.L.} = \lim_{h\to 0}(b_n + \cos\pi(2n-h)) = b_n + 1$$

$$\text{R.H.L.} = \lim_{h\to 0}(a_n + \sin\pi(2n+h)) = a_n$$

For continuity, $b_{n+1} = a_n$

Again, at $x = 2n + 1$

$$\text{L.H.L.} = \lim_{h\to 0}(a_n + \sin(\pi(2n+1-h))) = a_n$$

$$\text{R.H.L.} = \lim_{h\to 0}(b_{n+1} + \cos(\pi(2n+1+h))) = b_{n+1} - 1$$

Also, $f(2n + 1) = a_n$

For continuity, we require $b_n + 1 = a_n$

$\therefore \quad a_n - b_n = 1$

Also, $a_n = b_{n+1} - 1$

$\therefore \quad a_{n-1} - b_n = -1$

7. (b, c) : We have, $f(x) = x^3 + 3x + 2$,

$g(f(x)) = x$, $h(g(g(x))) = x$

Now, $f'(x) = 3x^2 + 3$

Again, $g'(2) = \dfrac{1}{f'(0)} = \dfrac{1}{3}$ $(\because\ f(0) = 2)$

Now, $h(g(g(x))) = x$

Differentiating with respect to x, we get

$$h'(g(g(x))) = \frac{1}{g'(g(x))\,g'(x)}$$

Now, $g(g(x)) = 1$

We have, $g(x) = f(1) = 6$

$f(6) = 236$

$$\therefore\ h'(1) = \frac{1}{g'(6)g'(236)} = \frac{1}{\frac{1}{6}\cdot\frac{1}{111}} = 666$$

Solving, $g(g(x)) = 0$ means $g(x) = g^{-1}(0)$.

$\Rightarrow \quad g(x) = 2$

$\therefore \quad x = g^{-1}(2) = f(2) = 16$

$\therefore \quad h(0) = 16$

Again, $h(g(g(x))) = x$

Put x to $f(x)$, then $h(g(g(f(x)))) = f(x)$

$\Rightarrow \quad h(g(x)) = f(x)$

$\therefore \quad h(g(3)) = f(3) = 38$

8. (3) : $\dfrac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2} = \dfrac{x\bar{x}+y\bar{y}+z\bar{z}}{|a|^2+|b|^2+|c|^2}$

$$= \frac{(a+b+c)(\bar{a}+\bar{b}+\bar{c}) + (a+b\omega+c\omega^2)(\bar{a}+\bar{b}\omega^2+\bar{c}\omega) + (a+b\omega^2+c\omega)(\bar{a}+\bar{b}\omega+\bar{c}\omega^2)}{|a|^2+|b|^2+|c|^2}$$

$$= \frac{3\left(|a|^2+|b|^2+|c|^2\right) + \bar{a}b\cdot 0 + a\bar{b}\cdot 0 + (b\bar{c}+c\bar{b}+a\bar{c}+\bar{a}c)\times 0}{|a|^2+|b|^2+|c|^2}$$

$$= \frac{3\left(|a|^2+|b|^2+|c|^2\right)}{|a|^2+|b|^2+|c|^2} = 3$$

9. (0) : Given $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$

Integrating factor $= e^{\int g'(x)dx} = e^{g(x)}$

Solution is

$$y(x)e^{g(x)} = \int e^{g(x)}g(x)g'(x)dx + k$$

Let $g(x) = v$, then

$$y(x)e^{g(x)} = \int e^v v\,dv + k = ve^v - e^v + k$$

Thus, $y(x) = (g(x) - 1) + ke^{-g(x)}$

$x = 0$, $y(0) = (g(0) - 1) + ke^{-g(0)} \Rightarrow k = 1$

Now, $y(2) = (g(2) - 1) + e^{-g(2)}$

Thus, $y(2) = 0$

10. (1) : $\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = \sin^{-1}\left(\dfrac{\sin\theta}{\sqrt{\sin^2\theta+\cos 2\theta}}\right)$

$$= \sin^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right) = \sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

As $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$,

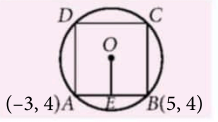
$$\text{Thus, } f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right) = \sin\left(\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right)$$

$= \sin(\sin^{-1}(\tan\theta)) = \tan\theta$

$$\text{Now, } \frac{d}{d(\tan\theta)}(f(\theta)) = 1$$

11. (32) : Let E be the midpoint of AB

$E = (1, 0)$



Let O be the centre of the circle.

AB is parallel to the x-axis.

$\therefore$ EO is parallel to the y-axis.

$\therefore$ Equation of EO is equal to $x = 1$

Since, the centre O lies on $4y = x + 7$

$4y = 1 + 7$

$\therefore \quad y = 2$

$\therefore$ centre is $(1, 2)$, $OE = 2$.

$\therefore \quad BC = 4$, $AB = 8$

Area of the rectangle $ABCD = AB \times BC$

$= 8 \times 4 = 32$ sq. units.

12. (4) : $\det A = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$

$= \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ 0 & (2k+1) & -(2k+1) \end{vmatrix}$ $[R_3 \to R_3 + R_2]$

$= (2k+1)\begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ 0 & 1 & -1 \end{vmatrix} = (2k+1)\begin{vmatrix} 2k-1 & 4\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1-2k & -2k \\ 0 & 0 & -1 \end{vmatrix}$ $[C_2 \to C_2 + C_3]$

$= (2k+1)^3$ (By expansion)

$\det B = 0$ ($\because B$ is skew symmetric matrix of odd order)

$\det(\text{adj } A) = (\det A)^{n-1} = \{(2k+1)^3\}^2 = (2k+1)^6$

Now, $(2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10 \Rightarrow 2k = 9$.

Thus, $[k] = 4$.

13. (7) : $T_n = {}^nC_3$

$\therefore \quad T_{n+1} - T_n = 21 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 21 \Rightarrow {}^nC_2 = 21$

$\Rightarrow \quad \frac{1}{2}n(n-1) = 21 \Rightarrow n^2 - n - 42 = 0$

$\Rightarrow \quad (n-7)(n+6) = 0$

As $n \geq 1$, we get $n = 7$.

14. (c) : $\alpha_1 = {}^6C_3 \cdot {}^5C_2 = 200$

$\alpha_2 = {}^6C_1 \cdot {}^5C_1 + {}^6C_2 \cdot {}^5C_2 + {}^6C_3 \cdot {}^5C_3 + {}^6C_4 \cdot {}^5C_4 + {}^6C_5 \cdot {}^5C_5$

$= 461$

$\alpha_3 = {}^{11}C_5 - {}^6C_5 - {}^6C_4 \cdot {}^5C_1 = {}^{11}C_5 - 6 - 75$

$= {}^{11}C_5 - 81 = 381$

G_1 includes : ${}^4C_1 \cdot {}^5C_2 + {}^4C_2\,{}^5C_1 + {}^4C_3 = 74$

M_1 includes : ${}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 34$

G_1 and M_1, both not included;

${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2 = 81$

$\therefore$ The total number of ways (α_4)

$= 74 + 34 + 81 = 189$

15. (a) : Let $F \equiv (4t^2, 8t)$

$\Delta EFG = \frac{1}{2}(3-4t)4t^2 \quad \frac{dA}{dt} = 12t - 24t^2$

$\therefore \quad t = 0$ (minima) ; $t = 1/2$ (maxima)

$m = \frac{8t-3}{4t^2-0} = \frac{4-3}{1} = 1$; $(\Delta EFG)_{\max} = \frac{6}{4} - 1 = \frac{1}{2}$

Also $y_0 = 8t = 4$ and $y_1 = 4t = 2$.

16. (a)

17. (b) : (P) $\cos x + \sin x = 1 \ \forall \ x \in \left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

So, $x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in Z$

$\Rightarrow \quad x = 2n\pi, \quad x = 2n\pi + \frac{\pi}{2}, n \in Z$

Thus, $x = 0, \frac{\pi}{2}$ are 2 solutions in the given range.

(Q) $\sqrt{3}\tan 3x = 1 \ \forall \ x \in \left[\frac{-5\pi}{18}, \frac{5\pi}{18}\right]$

$\Rightarrow \quad \tan 3x = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6} \Rightarrow 3x = n\pi + \frac{\pi}{6}$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

	5	1						2		2					6	9		
8			9				5				6			2		5	8	
4				8										5		7	2	
	9				1								2		7			
		8				9						1			9			
			8				4				4			1				
				7				4		9			2					
	1				4			8		5			3			8		
3						5	1				8			3	2			9
									3		8							
								1	6			3						
									1		4							
9						8	2	6		4			5	3				
	3				2			5		3		8			3			6
				8				3				1				2		9
			8			4	3						9				2	
		3				5								2			9	
	8				4										1			5
3				2												9		
7			5				9					1				6		3
	2	8						4		6							1	4

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$\Rightarrow \quad x = (6n+1)\frac{\pi}{18}, n \in Z$

Thus, $x = -\frac{5\pi}{18}, \frac{\pi}{18}$ are 2 solutions in the given range.

(R) $2\cos 2x = \sqrt{3} \quad \forall\ x \in \left[\frac{-6\pi}{5}, \frac{6\pi}{5}\right]$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \quad 2x = 2n\pi \pm \frac{\pi}{6}, n \in Z \Rightarrow \quad x = n\pi \pm \frac{\pi}{12}$

So, $x = \pm\frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}$ are 6 solutions in the given range.

(S) $\sin x - \cos x = 1 \quad \forall\ x \in \left[\frac{-7\pi}{4}, \frac{7\pi}{4}\right]$

$\Rightarrow \quad \cos x - \sin x = -1 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$\Rightarrow \quad x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}, n \in Z$

$\Rightarrow \quad x = 2n\pi + \frac{\pi}{2}, \ 2n\pi - \pi, n \in Z$

So, $x = \frac{\pi}{2}, -\frac{3\pi}{2}, \pi, -\pi$ are 4 solutions in the given range.

PAPER-II

1. (b) : $\int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$

$= \int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx + \int_1^2 (1+\cos^8 x)(ax^2+bx+c)dx$

$= \int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx$

$\therefore \int_1^2 (1+\cos^8 x)(ax^2+bx+c)dx = 0 \quad ...(i)$

Let $f(x) = \int_1^x (1+\cos^8 t)(at^2+bt+c)dt$

Then, $f(1) = 0$ and $f(2) = 0$ by (i)

Since, $f(x)$ is continuous on $[1, 2]$ and differentiable on $(1, 2)$ and $f(1) = f(2)$

$\therefore$ f satisfies all the conditions of Rolle's theorem

$\therefore \quad \exists\ \alpha \in (1, 2)$ such that $f'(\alpha) = 0$

$\Rightarrow \quad (1 + \cos^8\alpha)(a\alpha^2 + b\alpha + c) = 0$

$\Rightarrow \quad a\alpha^2 + b\alpha + c = 0 \qquad (\because\ 1 + \cos^8\alpha \neq 0)$

$\therefore$ The equation $ax^2 + bx + c = 0$ has at least one root in $(1, 2)$.

2. (c) : $\lim_{x\to a}\left[\frac{g(x)f(a) - g(a)f(x)}{x-a}\right]$

$= \lim_{x\to a}\left[f(a)\left\{\frac{g(x)-g(a)}{x-a}\right\} - g(a)\left\{\frac{f(x)-f(a)}{x-a}\right\}\right]$

$= [f(a)g'(a) - g(a)f'(a)] = (2)(2) - (-1)(1) = 5$

3. (b) : The desired area

$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$

Now, $\frac{1+\sin x}{\cos x} = \frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}$

$= \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}} = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

The expression for area reduces to

$\int_0^{\pi/4}\left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} - \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}}\right)dx = \int_0^{\pi/4}\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$

Let $\tan\frac{x}{2} = t$, the expression $= \int_0^{\pi/4}\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$

$= \int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

4. (b) : $\left.\begin{matrix}\sigma_1^2 = 4\\ \sigma_2^2 = 5\end{matrix}\right\}\begin{matrix}\bar{x} = 2\\ \bar{y} = 4\end{matrix}$

We have, $\frac{\sum x_i}{5} = 2 \Rightarrow \sum x_i = 10$

Similarly, $\Sigma y_i = 20$

$\sigma_1^2 = \left(\frac{1}{5}\sum x_i^2\right) - \bar{x}^2 \Rightarrow 4 = \frac{1}{5}\sum x_i^2 - 4$

$\Rightarrow \quad \frac{1}{5}\sum x_i^2 = 8 \ \therefore \ \sum x_i^2 = 40$

$$\sigma_2^2=\left(\frac{1}{5}\sum y_i^2\right)-\bar{y}^2 \Rightarrow 5=\frac{1}{5}\sum y_i^2-16$$

$$\Rightarrow \quad \frac{1}{5}\sum y_i^2=21. \quad \therefore \quad \sum y_i^2=105$$

$$\sigma^2=\frac{1}{10}\left(\sum x_i^2+\sum y_i^2\right)-\left(\frac{\bar{x}+\bar{y}}{2}\right)^2$$

$$=\frac{1}{10}(40+105)-9=\frac{145-90}{10}=\frac{55}{10}=\frac{11}{2}$$

5. (b, c) : Equation of curve is $xy = 1$

$$\Rightarrow \quad y+x\cdot\frac{dy}{dx}=0 \quad \text{or} \quad \frac{dy}{dx}=\frac{-y}{x}$$

If the line $ax + by + c = 0$ is normal to the curve at (x', y'), then the slope of the normal $= \frac{x'}{y'}$ $\therefore$ $\frac{x'}{y'}=\frac{-a}{b}$

$\because \quad x'y'=1 \Rightarrow \frac{x'}{y'}$ is positive $\Rightarrow \frac{-a}{b}$ is +ve

$\therefore$ Either $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

6. (a, c, d) : As $\tan(2\pi - \theta) > 0$

and $-1<\sin\theta<-\frac{\sqrt{3}}{2}, \theta\in(0,2\pi)$

Hence, $\frac{3\pi}{2}<\theta<\frac{5\pi}{3}$

Now, $2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1$

$\Rightarrow$ $2\cos\theta(1 - \sin\varphi) = 2\sin\theta\cos\varphi - 1$

$\Rightarrow$ $2\cos\theta + 1 = 2\sin(\theta + \varphi)$

As $\theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right), 1<2\sin(\theta+\varphi)<2$

$$\Rightarrow \theta+\varphi\in\left(\frac{\pi}{6},\frac{5\pi}{6}\right)\text{or}\,(\theta+\varphi)\in\left(\frac{13\pi}{6},\frac{17\pi}{6}\right)$$

We have, $\varphi\in\left(-\frac{3\pi}{2},-\frac{2\pi}{3}\right)\cup\left(\frac{2\pi}{3},\frac{7\pi}{6}\right)$.

7. (b, c) : $(x - \alpha)^2 + (y - \alpha)^2 = r^2$

$\Rightarrow$ $x^2 + y^2 - 2\alpha x - 2\alpha y + 2\alpha^2 - r^2 = 0$ ...(i)

Differentiating (i) w.r.t. x, we get

$2x + 2yy' - 2\alpha - 2\alpha y' = 0$ ...(ii)

$$\therefore \quad \alpha=\frac{x+yy'}{1+y'} \qquad \text{...(iii)}$$

Differentiating (ii), we get

$1 + y'^2 + yy'' - \alpha y'' = 0$

$$\Rightarrow \quad 1+y'^2+yy''-\frac{x+yy'}{(1+y')}y''=0 \qquad \text{[From (iii)]}$$

Collecting terms, we get

$(y - x)y'' + (1 + y' + y'^2)y' + 1 = 0$

Now compare with given differential equation.

$P + Q = y - x + 1 + y' + y'^2$

8. (4) : Let $x - 2 > 0$ *i.e.* $x > 2$

Then, $(x - 2)^2 + x - 2 - 2 = 0$

$x^2 - 4x + 4 + x - 4 = 0$

$x^2 - 3x = 0$ $\therefore$ $x = 0$ (Not possible)

Possible solution : $x = 3$

Now, $x - 2 < 0$ *i.e.* $x < 2$

Then, $(x - 2)^2 - (x - 2) - 2 = 0$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$\Rightarrow$ $(x - 1)(x - 4) = 0$

$x = 1$ (Accepted) $x = 4$ (Not possible)

Hence, roots are 1 and 3.

$\therefore$ Sum $= 1 + 3 = 4$

9. (5) : Let vectors be given as

$\vec{a}=(1,1,1); \qquad \vec{a}'=(-1,-1,-1)$

$\vec{b}=(-1,1,1); \qquad \vec{b}'=(1,-1,-1)$

$\vec{c}=(1,-1,1); \qquad \vec{c}'=(-1,1,-1)$

$\vec{d}=(1,1,-1); \qquad \vec{d}'=(-1,-1,1)$

$\vec{a}$ and $\vec{a}'$ with any of the remaining six will produce a set of three coplanar vectors.

Number of such choices $= 4 \times 6 = 24$

The number of non coplanar vectors

$= {}^8C_3 - 24 = 56 - 24 = 32 = 2^5$ $\therefore$ $p = 5$

10. (4) : Let $t=\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}}....}$; $t > 0$

$$t^2=4-\frac{1}{3\sqrt{2}}t \quad \Rightarrow \quad t^2+\frac{1}{3\sqrt{2}}t-4=0 \quad \Rightarrow \quad t=\frac{8}{3\sqrt{2}}$$

The required answer

$$=6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right)=6+\log_{\frac{3}{2}}\frac{4}{9}=6-2=4$$

Recipe for Success

"Success is not how high you have climbed, but how you make a positive difference to the world."

–Roy T. Bennett

11. (1) : Let

$$\log y_n = \frac{1}{n}\left\{\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+\ldots+\log\left(1+\frac{n}{n}\right)\right\}$$

$$\therefore \lim_{n\to\infty} \log y_n = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$= \int_0^1 \log(1+x)dx = \int_1^2 \log x\, dx$$

$$= \left[x\log x - x\right]_1^2 = 2\log 2 - 1 = \log\frac{4}{e}$$

$\therefore$ $L = 4/e$ Thus, $[L] = 1$.

12. (0.5) : Given, E_2 : sum of entries of A $= 7$

$\therefore$ $n(E_2) = {}^9C_2 = 36$

Now, $E_1 \cap E_2$ will be the set of those matrices, where both zeros lie in same row or in the same column.

$\therefore$ $n(E_1 \cap E_2) = 2\times {}^3C_2 \times {}^3C_2 = 18$

Now, $P(E_1/E_2) = \dfrac{n(E_1\cap E_2)}{n(E_2)} = \dfrac{18}{36} = \dfrac{1}{2} = 0.5$

13. (15) : $\displaystyle\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ is the coefficient of x^m in the expansion of binomial expression $(1 + x)^{10}\,(x + 1)^{20}$

$\Rightarrow \displaystyle\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ = coefficient of x^m in the binomial expression $(1 + x)^{30}$.

$\Rightarrow \displaystyle\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i} = {}^{30}C_m$

Now, ${}^{30}C_m$ is maximum when $m = \dfrac{30}{2} = 15$

(Since, 30 is an even positive integer)

14. (43) : The required probability is

$$\frac{4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{5!} = \frac{9}{120} = \frac{3}{40} = \frac{m}{n}$$

$\therefore$ $m + n = 43$

15. (60) : Total cases = 5!

Lets count the number of favourable ways.

$$\left.\begin{matrix}2&4&1&3&5\\2&5&3&1&4\end{matrix}\right\}\text{3 ways} \times 2 = 6$$

$$\left.\begin{matrix}1&3&5&2&4\\1&4&2&5&3\end{matrix}\right\}\text{2 ways} \times 2 = 4$$

$$\left.\begin{matrix}3&1&5&2&4\\3&1&4&2&5\end{matrix}\right\}\text{2 ways} \times 2 = 4$$

$\therefore$ Total favourable ways = 14

Required probability $\dfrac{14}{5!} = \dfrac{7}{60} = \dfrac{7}{p}$

$\therefore$ $p = 60$

16. (2) : A tangent to the hyperbola

$\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ is $\dfrac{x\sec\theta}{3} - \dfrac{y\tan\theta}{2} = 1$

As it touches the circle $x^2 + y^2 - 8x = 0$,

i.e. $(x - 4)^2 + y^2 = 16$, then we have,

$$\frac{\left|\frac{4\sec\theta}{3}-1\right|}{\sqrt{\frac{\sec^2\theta}{9}+\frac{\tan^2\theta}{4}}} = 4 \Rightarrow \frac{|4\sec\theta-3|}{\sqrt{4\sec^2\theta+9\tan^2\theta}} = 2$$

$\Rightarrow$ $16\sec^2\theta - 24\sec\theta + 9 = 4\{4\sec^2\theta + 9\sec^2\theta - 9\}$

$(\because \sec^2\theta = \tan^2\theta + 1)$

$\Rightarrow$ $12\sec^2\theta + 8\sec\theta - 15 = 0$

$\Rightarrow \sec\theta = \dfrac{5}{6}, -\dfrac{3}{2}$

So, $\tan\theta = \dfrac{\sqrt{5}}{2}$ $\left(\text{as } \sec\theta \neq \dfrac{5}{6}\right)$

Thus, equation of tangent is $2x - \sqrt{5}y + 4 = 0$.

$\therefore$ $b/a = 2$

17. (38) : The point of intersection is given by solving

$\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ with $x^2 + y^2 - 8x = 0$

i.e., $\dfrac{x^2}{9} + \dfrac{x^2 - 8x}{4} = 1 \Rightarrow 4x^2 + 9x^2 - 72x = 36$

$\Rightarrow$ $13x^2 - 72x - 36 = 0 \Rightarrow (13x + 6)(x - 6) = 0$

$\Rightarrow x = -\dfrac{6}{13}$ or $x = 6$

So, $x = 6$ $\left(\text{as } x \neq \dfrac{-6}{13}\right)$

$A = (6, 2\sqrt{3}), B = (6, -2\sqrt{3})$

The circle with AB as diameter is

$(x-6)(x-6) + (y+2\sqrt{3})(y-2\sqrt{3}) = 0$

$\Rightarrow$ $(x-6)^2 + y^2 = (2\sqrt{3})^2 \Rightarrow x^2 + y^2 - 12x + 24 = 0$

$\therefore$ $a + b + c + d = 38$

CUET (UG)

Between 8th May to 1st June 2025

PRACTICE PAPER 2025

Section II of CUET (UG) is domain specific. Section - A is compulsory, however students have to choose between Section - B_1 and Section - B_2.

Time Allowed : 60 Minutes **Maximum Marks :** 250

SECTION - A

1. If for the non-singular matrix A, $A^2 = I$, then find A^{-1}.
(a) A (b) I
(c) O (d) None of these

2. Evaluate : $\int_0^{2\pi} \sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$
(a) $-2\sqrt{2}$ (b) -2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

3. If a matrix A is both symmetric and skew-symmetric, then
(a) A is a diagonal matrix
(b) A is a zero matrix
(c) A is a scalar matrix
(d) A is a square matrix

4. If $A=\begin{pmatrix} x & 2 \\ 2 & x \end{pmatrix}$ and det $(A^2) = 25$, then x is equal to
(a) ± 3 (b) ± 1 (c) ± 2 (d) ± 4

5. The solution of $\frac{dy}{dx}=1+x+y^2+xy^2$, when $y = 0$, $x = 0$ is
(a) $\tan\left(x-\frac{x^2}{2}\right)$ (b) $\tan\left(\frac{x^2}{2}+1\right)$
(c) $\cot\left(x+\frac{x^2}{2}\right)$ (d) $\tan\left(x+\frac{x^2}{2}\right)$

6. Which of the following is not a vertex of the feasible region bounded by the inequalities $2x + 3y \le 6$, $5x + 3y \le 15$ and $x, y \ge 0$?
(a) (0, 2) (b) (0, 0)
(c) (3, 0) (d) (0, 5)

7. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to
(a) $\frac{\sin x - e^x}{(\cos x + e^x)^2}$ (b) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$
(c) $\frac{\sin x - e^x}{(\cos x + e^x)}$ (d) $(-\sin x + e^x)^{-1}$

8. Consider the linear programming problem
Max. $Z = 4x + y$
Subject to $x + y \le 50$; $x + y \ge 100$; $x, y \ge 0$
The max. value of Z is
(a) 0 (b) 50
(c) 100 (d) does not exist

9. Solve the following system of equations :
$3x - y + z = 5$, $2x - 2y + 3z = 7$, $x + y - z = -1$
(a) $x = -1, y = -1, z = 1$
(b) $x = 1, y = 1, z = 1$
(c) $x = 1, y = -1, z = 1$ (d) $x = 1, y = -1, z = -1$

10. If $y = x^3 + x^2 + x + 1$, then y
(a) has a local minimum
(b) has a local maximum
(c) neither have a local minimum nor local maximum
(d) None of these

11. The random variable X can take only the values 0, 1, 2. Given that, $P(X = 0) = P(X = 1) = p$ and that $\sum x_i^2 p_i = \sum x_i p_i$, find the value of p.
(a) $\frac{1}{5}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$

12. If $y = \log_{10} x + \log_e x$, then $\frac{dy}{dx} =$
(a) $\frac{\log_{10} e}{x}+\frac{1}{x}$ (b) $\log_{10} e + x$
(c) $\log_e 10+\frac{1}{x}$ (d) none of these

13. The order of the differential equation whose general solution is given by

$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$

where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is

(a) 5 (b) 4 (c) 3 (d) 2

14. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

Then, match the terms of **List-I** with their respective values in **List-II**.

List-I		List-II	
(A)	The value of k =	(I)	0.55
(B)	P(you study atleast 2 hours) =	(II)	0.3
(C)	P(you study exactly 2 hours) =	(III)	0.75
(D)	P(you study atmost 2 hours) =	(IV)	0.15

Choose the **correct** answer from the options given below :

(a) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(b) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(c) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(d) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

15. $\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x} dx$ is equal to

(a) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(b) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

(c) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(d) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

SECTION - B1 (MATHEMATICS)

16. The value of the determinant

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z - a^{-z})^2 & 1 \end{vmatrix}$$

where $a > 0$ and $x, y, z \in R$ is

(a) $a^x a^y a^z$ (b) 1
(c) 0 (d) $a^x + a^y + a^z$

17. $\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$ $(x > 0)$ is

(a) $\tan^{-1}\left(x + \frac{1}{x}\right) + c$ (b) $\tan^{-1}\left(x - \frac{1}{x}\right) + c$

(c) $\log_e \left|\frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1}\right| + c$ (d) $\log_e \left|\frac{x - \frac{1}{x} - 1}{x - \frac{1}{x} + 1}\right| + c$

18. The values of k for which the system $(k + 1)x + 8y = 0$; $kx + (k + 3)y = 0$ has unique solution, are

(a) 3, 1 (b) −3, 1
(c) 3, −1 (d) −3, −1

19. The solution of differential equation $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$ is

(a) $(e^y + 1)\sin x = c$ (b) $e^y \sin x = c$
(c) $(e^y - 1)\sin x = c$ (d) none of these

20. In $\mathbb{R}$, a relation p is defined as follows : $\forall a, b \in \mathbb{R}$. $a\ p\ b$ holds iff $a^2 - 4ab + 3b^2 = 0$. Then

(A) p is equivalence relation
(B) p is only symmetric
(C) p is only reflexive
(D) p is only transitive

Choose the **correct** answer from the options given below :

(a) (A) and (C) only
(b) (B) only
(c) (C) only
(d) (A), (C) and (D) only

21. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag without replacement. Then match **List-I** with **List-II**.

List-I		List-II	
(A)	Probability that all the four balls are black is equal to	(I)	14/33
(B)	If the bag contains 10 black and two white balls, then the probability that all four balls are black is equal to	(II)	1/5
(C)	If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	(III)	70/429
(D)	Probability that two balls are black and two are white is	(IV)	13/165

(a) (A)-(II), (B)-(I), (C)-(III), (D)-(II)
(b) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(c) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(d) (A)-(II), (B)-(I), (C)-(III), (D)-(I)

22. Let $f: R \to R$ be differentiable at $x = 0$. If $f(0) = 0$ and $f'(0) = 2$, then the value of

$$\lim_{x\to 0}\frac{1}{x}[f(x)+f(2x)+f(3x)+\ldots+f(2015x)] \text{ is}$$

(a) 2015 (b) 0
(c) 2015×2016 (d) 2015×2014

23. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then the value of a and b are

(a) $a = 1, b = -1$ (b) $a = -1, b = 1$
(c) $a = 1, b = 1$ (d) $a = -1, b = -1$

24. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj } 2A))| = (16)^n$, then n is equal to

(a) 9 (b) 10 (c) 12 (d) 8

25. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0, f > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to

(a) $2f(1)$ (b) $f(1)$ (c) $4f(2)$ (d) $4f(1)$

26. The area (in sq. units) of the parallelogram whose diagonals are along the vectors

$8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is

(a) 52 (b) 26 (c) 65 (d) 20

27. For the mapping $f : R - \{1\} \to R - \{2\}$, given by $f(x) = \frac{2x}{x-1}$, then

(A) f is one-one but not onto
(B) f is onto but not one-one
(C) f is neither one-one nor onto
(D) f is both one-one and onto

Choose the **correct** answer from the options given below:

(a) (A), (B) and (D) only
(b) (A) and (B) only
(c) (A) only
(d) (D) only

28. If A and B are two independent events, then the probability of occurrence of atleast one of A and B is given by

(a) $1 - P(A)\,P(B)$ (b) $1 - P(A)\,P(B')$
(c) $1 - P(A')\,P(B')$ (d) $1 - P(A')\,P(B)$

29. A line makes the same angle θ, with both the x-axis and z-axis. If it makes angle β, with the y-axis such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals to

(a) $\frac{5}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

30. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is

(A) $7A - I$ (B) $7A$
(C) $I - 7A$ (D) $7A + I$

Choose the **correct** answer from the options given below:

(a) (A), (B) and (D) only
(b) (A) and (B) only
(c) (A) only
(d) (D) only

31. Domain of $\cos^{-1}[x]$ (where $[\cdot]$ denotes G.I.F.) is

(a) $[-1, 2]$ (b) $[-1, 2)$
(c) $(-1, 2]$ (d) None of these

32. A particle is moving along the curve $y = 8x + \cos y$, $0 \le y \le \pi$. If at a point the ordinate is changing 4 times as fast as the abscissa, then the coordinates of the point are

(a) $\left(\frac{\pi}{16}, \frac{\pi}{2}\right)$ (b) $\left(\frac{-1}{8}, 0\right)$

(c) $\left(\frac{1}{8}, 0\right)$ (d) $\left(\frac{-\pi}{2}, \frac{-\pi}{16}\right)$

33. The function $f(x) = \cot^{-1}x + x$ increases in the interval

(a) $(1, \infty)$ (b) $(-1, \infty)$

(c) $(0, \infty)$ (d) $(-\infty, \infty)$

34. Match **List-I** with **List-II**.

List I		List II	
(A)	If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}+2\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is	(I)	90°
(B)	If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-2\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is	(II)	obtuse
(C)	If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is	(III)	0°
(D)	Angle between $\vec{a}\times\vec{b}$ and a vector perpendicular to the vector $\vec{c}\times(\vec{a}\times\vec{b})$ is	(IV)	acute

Choose the **correct** answer from the options given below :

(a) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(b) (A) – (II), (B) – (IV), (C) – (I), (D) – (III)

(c) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(d) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

35. If $P(A)=\frac{1}{4}, P(B)=\frac{1}{5}$ and $P(A\cap B)=\frac{1}{8}$ then $P\left(\frac{A^C}{B^C}\right)=$

(a) $\frac{21}{32}$ (b) $\frac{25}{32}$ (c) $\frac{27}{32}$ (d) $\frac{29}{32}$

36. Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x^2 - 1|$, then

(A) f has a local minima at $x = \pm 1$ but no local maxima

(B) f has a local maxima at $x = 0$

(C) f has a local minima at $x = \pm 1$

(D) f has neither any local maxima nor any local minima

Choose the **correct** answer from the options given below :

(a) (A) and (B) only

(b) (B) and (C) only

(c) (A), (B) and (C) only

(d) (B), (C) and (D) only

37. Let $\vec{a}$ and $\vec{b}$ be two vectors of equal magnitude inclined at an angle θ, then $|\vec{a}|\sin\frac{\theta}{2}$ is equal to

(a) $\frac{|\vec{a}-\vec{b}|}{2}$ (b) $\frac{|\vec{a}+\vec{b}|}{2}$ (c) $|\vec{a}-\vec{b}|$ (d) $|\vec{a}+\vec{b}|$

38. If $f(x)=\sum_{n=0}^{\infty}\frac{x^n}{n!}(\log a)^n$, then at $x = 0$, $f(x)$

(a) has no limit (b) is discontinuous

(c) is continuous but not differentiable

(d) is differentiable

39. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $\frac{x}{y}$

40. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

41. The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is

(a) $\left[-\frac{1}{2}, \infty\right) - \{0\}$ (b) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

(c) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$ (d) $\left(-\frac{1}{2}, \infty\right) - \{0\}$

42. Match **List-I** with **List-II** :

List-I		List-II	
(A)	Area under the curve $y = x^2$ between the lines $x = 1$ and $x = 3$ is	(I)	$\frac{7}{3}$ sq. units
(B)	Area enclosed between the x-axis and the curve $y = x^2 - 1$ is	(II)	$\frac{26}{3}$ sq. units
(C)	Area enclosed between the x-axis, y-axis and the curve $y^2 = x + 1$ and which lies above the x-axis is	(III)	$\frac{4}{3}$ sq. units
(D)	Area bounded by the curve $y^2 = x$, the y-axis and the lines $y = 1$, $y = 2$ is	(IV)	$\frac{2}{3}$ sq. unit

Choose the **correct** answer from the options given below:

(a) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(b) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(c) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(d) (A)–(I), (B)–(III), (C)–(II), (D)–(IV)

43. Which of the following is true?

(A) The probability that out of 10 persons, all born in April, at least two have the same birthday is $\frac{{}^{30}C_{10}}{30^{10}}$.

(B) If A and B are mutually exclusive events, then they will be independent also.

(C) If A and B are two independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

(D) Let $P(A) > 0$ and $P(B) > 0$. Then A and B can be both mutually exclusive and independent.

Choose the **correct** answer from the options given below:

(a) (C) and (D) only (b) (B) and (D) only

(c) (C) only (d) (A) and (B) only

44. Which of the following is true?

(A) If a straight line makes the angles 60°, 45° and α with x, y and z axes respectively, then $\sin^2\alpha = 3/4$.

(B) The angles between the lines with direction ratios 4, –3, 5 and 3, 4, 5 is $\frac{\pi}{6}$.

(C) The shortest distance between the lines $\frac{x}{-1} = \frac{y}{1} = \frac{z}{1}$ and $\frac{x-3}{0} = \frac{y+3}{1} = \frac{z-3}{-1}$ is $\sqrt{6}$ units

Choose the **correct** answer from the options given below:

(a) (B) only (b) (A) and (C) only

(c) (A) only (d) (B) and (C) only

45. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(a) $\frac{3}{4}$ (b) $\frac{52}{867}$ (c) $\frac{39}{50}$ (d) $\frac{22}{425}$

46. Solve : $x \sin x \frac{dy}{dx} + (x + x\cos x + \sin x)\, y = \sin x$, $y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi}$

(a) $y(x) = \frac{x - \sin x}{x(1 - \cos x)}$ (b) $y(x) = \frac{x + \sin x}{x(1 - \cos x)}$

(c) $y(x) = \frac{x \sin x}{\cos x - 1}$ (d) None of these

47. The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is

(a) $8\sqrt{3}$ sq. units (b) 8 sq. units

(c) $2\sqrt{3}$ sq. units (d) $3\sqrt{3}$ sq. units

48. For L.P.P., maximize $z = 4x_1 + 2x_2$ subject to $3x_1 + 2x_2 \geq 9$, $x_1 - x_2 \leq 3$, $x_1 \geq 0$, $x_2 \geq 0$ has

(a) Infinite number of optimal solutions

(b) Unbounded solution

(c) No solution

(d) One optimal solution

49. Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely.

If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then find $P(E|F)$ and $P(F|E)$.

(a) $\frac{1}{4}, \frac{1}{13}$ (b) $\frac{1}{5}, \frac{1}{11}$ (c) $\frac{1}{4}, \frac{1}{12}$ (d) $\frac{1}{3}, \frac{1}{13}$

50. Match **List-I** with **List-II** :

List-I		List-II	
(A)	$\int_0^{\pi/2} \log(\tan x)\, dx$ is equal to	(I)	$\int_0^{\pi/2} \cos^2 x \sin x\, dx$
(B)	$\int_0^{\pi/2} \sin^2 x\, dx$ is equal to	(II)	$\int_0^{\pi/2} \log(\cot x)\, dx$
(C)	$\int_0^{\pi/2} \sin^3 x\, dx$ is equal to	(III)	$\int_0^{\pi/2} \cos^2 x\, dx$
(D)	$\int_0^{\pi/2} \sin^2 x \cos x\, dx$ is equal to	(IV)	$\int_0^{\pi/2} \sin x \sin 2x\, dx$

(a) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(b) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(c) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(d) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

SECTION - B2 (APPLIED MATHEMATICS)

51. Two vessels A and B contain milk and water in the ratio 7 : 5 and 17 : 7 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 5 : 3?

(a) 2 : 1 (b) 1 : 2 (c) 3 : 2 (d) 2 : 3

52. If A is a skew symmetric matrix, then A^{2021} is

(a) Row matrix (b) Symmetric matrix
(c) Column matrix
(d) Skew symmetric matrix

53. If the parabola $y = \alpha x^2 - 6x + \beta$ passes through the point (0, 2) and has its tangent at $x = \frac{3}{2}$ parallel to x-axis, then

(a) $\alpha = 2, \beta = -2$ (b) $\alpha = -2, \beta = 2$
(c) $\alpha = 2, \beta = 2$ (d) $\alpha = -2, \beta = -2$

54. 3^{16} mod 4 is equal to

(a) 1 (b) 2 (c) 12 (d) 64

55. $\int e^x \sqrt{e^x}\, dx =$

(a) $\frac{3}{2} e^x \sqrt{e^x} + C$ (b) $\frac{2}{3} e^x \sqrt{e^x} + C$

(c) $\frac{5}{2} e^{2x} \sqrt{e^x} + C$ (d) $\frac{2}{5} e^{2x} \sqrt{e^x} + C$

56. Match **List-I** with **List-II** :

List-I		List-II	
(A)	Average Cost (AC)	(I)	$\frac{d}{dx} C(x)$
(B)	Marginal Cost (MC)	(II)	$\frac{1}{x}(MC - AC)$
(C)	Marginal Average Cost (MAC)	(III)	$\frac{C(x)}{x}$
(D)	Marginal Revenues	(IV)	$\frac{d}{dx} R(x)$

(a) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)
(b) (A) – (I), (B) – (III), (C) – (IV), (D) – (II)
(c) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)
(d) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)

57. Find the values of a and b for which the following hold:

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

(a) $a = 1, b = 2$ (b) $a = -2, b = 3$
(c) $a = 1, b = -3$ (d) $a = -2, b = -3$

58. Let $I = \int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1 + 5^x}\, dx$. Then

(a) $I = \int_{-\pi/4}^{\pi/4} \tan^2 x\, dx$ (b) $2I = \int_{-\pi/4}^{\pi/4} \tan^2 x\, dx$

(c) $I = \int_{-\pi/4}^{\pi/4} \frac{1}{1 + 5^x}\, dx$

(d) $2I = \int_{-\pi/4}^{\pi/4} 5^x \tan^2 x\, dx$

59. Rakesh and Suresh invest in a business in the ratio 4 : 5. If 10% of the profit goes for charity purpose and Rakesh's share of profit is ₹ 18000. Find the total profit.

(a) ₹ 54000 (b) ₹ 45000
(c) ₹ 36000 (d) None of these

60. Match the **List-I** with **List-II** :

List-I		List-II	
(A)	21 mod (2)	(I)	8
(B)	(8 × 7) mod 6	(II)	1
(C)	12 mod 4	(III)	2
(D)	(9 + 23) mod 12	(IV)	0

(a) (A) – (I), (B) – (III), (C) – (IV), (D) – (II)
(b) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
(c) (A) – (I), (B) – (IV), (C) – (III), (D) – (II)
(d) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

61. Two number are selected at random (without replacement) from the first 5 positive integers. Let X denote the larger of the two numbers obtained. Find the mean of this distribution.

(a) 1 (b) 2 (c) 4 (d) 8

62. The differential equation of the family of parabola with focus at the origin and the x-axis as axis, is

(a) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$

(b) $y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$

(c) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$

(d) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

63. The mean and variance of a binomial distribution are 8 and 4 respectively. Find $P(X = 1)$.

(a) $1/2^8$ (b) $1/2^{12}$

(c) $1/2^6$ (d) $1/2^4$

64. If X is a poisson variable such that $3P(X=4)=\frac{1}{2}P(X=2)+P(X=0)$, then the mean of X is

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

65. Two pipes A and B together can fill a tank in 10 minutes. If pipe A take 15 minutes less than B to fill the tank alone, then find the time taken by pipe B to fill the tank alone.

(a) 30 min (b) 26 min

(c) 24 min (d) None of these

66. The degree and order of the differential equation $\frac{d^2y}{dx^2}=\sqrt[3]{1+\left(\frac{dy}{dx}\right)^2}$ respectively are

(a) 2 and 3 (b) 3 and 2

(c) 2 and 2 (d) 3 and 3

67. Which of the following factor does not affect EMI (Equated Monthly Installment)

(a) Principal borrowed (b) Rate of interest

(c) Market trend (d) Tenure of the loan

68. A boat can row upstream at 10 km/h and downstream at 18 km/h. Then

(A) Speed of boat in still water is 14 km/h.

(B) Speed of boat in still water is 8 km/h.

(C) Speed of the stream is 4 km/h.

(D) Speed of the stream is 7 km/h.

Which of the above statement is/are true?

(a) (A) and (C) only (b) (A) and (D) only

(c) (B) and (C) only (d) (B) and (D) only

69. Which of the following is test of adequacy of index numbers?

(a) Time reversal test

(b) Factor reversal test

(c) Circular test

(d) All of these

70. A is a real skew-symmetric matrix such that $A^2 + I = 0$, then AA' is equal to

(a) $A + I$ (b) A^2

(c) I (d) None of these

71. Match **List-I** with **List-II :**

List-I		List-II	
(A)	$\lvert x+3\rvert \geq 10, x\in$	(I)	$(-13, -3)$
(B)	$\frac{x-7}{x+3}>2, x\in$	(II)	$(-12, -5)$
(C)	$\frac{x-2}{x+5}>2, x\in$	(III)	$(-\infty, -13]\cup[7, \infty)$

(a) (A) – (I), (B) – (III), (C) – (II)

(b) (A) – (III), (B) – (I), (C) – (II)

(c) (A) – (II), (B) – (I), (C) – (III)

(d) (A) – (III), (B) – (II), (C) – (I)

72. If for a matrix A, $A^2 = A$ and $B = I–A$, then $AB + BA + I – (I – A)^2$ equals

(a) $A^2 – I$ (b) A

(c) I (d) None of these

73. Consider the following statements.

(A) Saving accounts opens for necessarily a specific purpose.

(B) Sinkings funds are set up for specific upcoming expenses.

(C) Sinking funds are meant for large unexpected expenses.

(D) Savings accounts are set up for long-term savings

Which of the given statements(s) is/are true?

(a) (D) only (b) (A), (B) and (D) only

(c) (B) and (D) only (d) (B), (C) and (D) only

74. Increase in the number of patients in the hospital due to heat stroke is

(a) Secular trend (b) Cyclical variation

(c) Seasonal variation (d) Irregular variation

75. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(X = 3) = 2P(X = 1)$ and $P(X = 2) = 0.3$. Then match the **List-I** with **List-II** and select the correct answer using the options given below :

List-I		List-II	
(A)	$P(X = 0)$ is	(I)	0.1
(B)	Variance of X is	(II)	0.4
(C)	Standard deviation of X is	(III)	1.41
(D)	$P(X = 1)$ is	(IV)	1.19

(a) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
(b) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)
(c) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)
(d) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

76. Find $\frac{d^2y}{dx^2}$ if $x = k(A - \sin A)$ and $y = k(1 - \cos A)$

(a) $-\frac{\cos ec^2 A/2}{2} \cdot \frac{1}{k(1-\cos A)}$

(b) $\frac{-\cos ec^2 A/2}{2\sin A}$ (c) $\frac{\cot A/2}{k(1-\cos A)}$

(d) None of these

77. A die is thrown 10 times. The probability that an odd number will come up at least once is

(a) $\frac{11}{1024}$ (b) $\frac{1013}{1024}$

(c) $\frac{1023}{1024}$ (d) $\frac{1}{1024}$

78. Which of the following is not a component of time series.

(a) Seasonal (b) Cyclical
(c) Irregular (d) Variance

79. Match **List-I** with **List-II** and select the correct answer using the options given below :

List-I		List-II	
(A)	The function $f(x) = \frac{x}{(1+x^2)}$ decreases in the interval	(I)	$(-\infty, \infty)$
(B)	The function $f(x) = \tan^{-1} x - x$ decreases in the interval	(II)	$(-\infty, 0)$
(C)	The function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ increases in the interval	(III)	$(0, \infty)$
		(IV)	$(-\infty, -1) \cup (1, \infty)$

(a) (A) – (IV), (B) – (I), (C) – (II)
(b) (A) – (II), (B) – (III), (C) – (IV)
(c) (A) – (I), (B) – (II), (C) – (III)
(d) (A) – (IV), (B) – (II), (C) – (III)

80. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ =

(a) $\frac{-4x}{1-x^4}$ (b) $\frac{1}{\sqrt{x^2+1}}$

(c) $\frac{-1}{1-x^4}$ (d) None of these

81. Price relative of rice is 110 in 2019 compared to 2018. If rice costs ₹ 16.50 per kg in 2019, what did it cost in 2018.

(a) ₹ 14 per kg (b) ₹ 14.5 per kg
(c) ₹ 15 per kg (d) ₹ 15.5 per kg

82. In what ratio must a shopkeeper mix two types of oranges worth ₹ 30 per kg and ₹ 45 per kg respectively so as to get a mixture at ₹ 40 per kg?

(a) 3 : 2 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

83. Find the remainder when
672 + 541 + 383 + 295 + 101 + 86 is divided by 3.

(a) 0 (b) 1
(c) 2 (d) Can't determined

84. The critical points for the function $f(x) = x^3 - 3x^2 + 3x + 4$ are

(a) 1, 1/3 (b) 1, 3 (c) 1 only (d) 3 only

85. If $y = \frac{\ln x}{x}$, then $\frac{d^2y}{dx^2}$ at $(x = 1)$ is equal to

(a) 3 (b) – 3 (c) 2 (d) –2

SOLUTIONS

1. (a) : Given, $A^2 = I$ ...(i)

Since, A is non-singular matrix

$\Rightarrow \quad |A| \neq 0$

So, A^{-1} exists.

Multiplying (i) by A^{-1} on both sides, we get

$A^{-1}(A^2) = A^{-1}(I)$

$\Rightarrow \quad (A^{-1}A)A = A^{-1} \Rightarrow IA = A^{-1} \quad (\because A^{-1}A = I)$

$\therefore \quad A^{-1} = A \quad (\because IA = A)$

2. (d) : We have, $\int_0^{2\pi} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

$$= \left[-2\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]_0^{2\pi} = -2\left(\cos\left(\frac{\pi}{4} + \pi\right) - \cos\frac{\pi}{4}\right)$$

$$= 2\cos\frac{\pi}{4} + 2\cos\frac{\pi}{4} = 2\sqrt{2}$$

3. (b) : Since, A is a symmetric matrix

$\Rightarrow \quad A^T = A$ ...(i)

Also, A is a skew-symmetric matrix

$\Rightarrow \quad A^T = -A$ ...(ii)

From (i) and (ii), we get

$A = -A \quad \Rightarrow \quad A = O$

Hence, A is zero matrix.

4. (a) : We have, $\det(A^2) = 25$

$\Rightarrow \quad [\det(A)]^2 = 25$

$\Rightarrow \quad \det(A) = \pm 5$

$\Rightarrow \quad \begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix} = \pm 5 \Rightarrow x^2 - 4 = \pm 5$

Now, $x^2 - 4 = -5$ gives $x^2 < 0$, we neglect this case.

Also, $x^2 - 4 = 5$

$\Rightarrow \quad x^2 = 9 \quad \Rightarrow \quad x = \pm 3$

5. (d) : We have, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \quad \frac{dy}{dx} = (1 + y^2)(1 + x)$

$\Rightarrow \quad \frac{dy}{1 + y^2} = (1 + x)dx$

Integrating both sides, we get

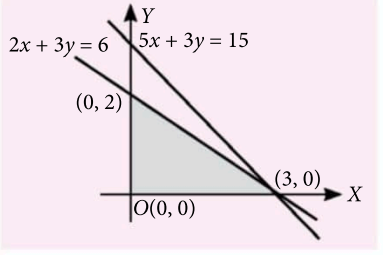
$\tan^{-1} y = x + \frac{x^2}{2} + c$ ...(i)

When, $y = 0, x = 0$

$\Rightarrow \tan^{-1}(0) = 0 + 0 + c \quad \therefore \quad c = 0$

Now, from (i), $\tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

6. (d) : Here (0, 2), (0, 0) and (3, 0) all are vertices of feasible region. Hence option (d) is correct.



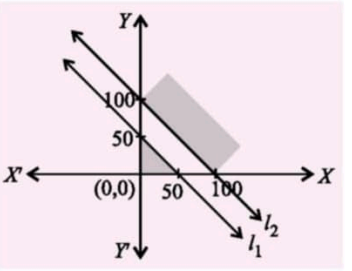
7. (b) : $y = \sin x + e^x$

$\Rightarrow \quad \frac{dy}{dx} = \cos x + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{\cos x + e^x}$

$\therefore \quad \frac{d^2x}{dy^2} = -\frac{1}{(\cos x + e^x)^2}[-\sin x + e^x]\frac{dx}{dy}$

$= -\frac{(e^x - \sin x)}{(\cos x + e^x)^2} \times \frac{1}{\cos x + e^x} = \frac{\sin x - e^x}{(\cos x + e^x)^3}$

8. (d) : Let $l_1 : x + y = 50$; $l_2 : x + y = 100$; $l_3 : x = 0$; $l_4 : y = 0$



Since, no feasible region determined, hence, no maximum value of Z exists.

9. (c)

10. (c) : Let $f(x) = y = x^3 + x^2 + x + 1$

Then, $f'(x) = 3x^2 + 2x + 1$.

For a maximum or minimum, we have

$f'(x) = 0 \Rightarrow 3x^2 + 2x + 1 = 0$

But, this equation gives imaginary values of x.

So, $f'(x) \neq 0$ for any real value of x.

Hence, $f(x)$ does not have a maximum or minimum.

11. (d) : Given that, $P(X = 0) = p$, $P(X = 1) = p$

Let $P(X = 2) = x$

$\because \quad \Sigma p_i = 1 \quad \Rightarrow \quad p + p + x = 1$

$\Rightarrow \quad x = 1 - 2p$ ...(i)

Also, $\Sigma x_i p_i = 0 \times p + 1 \times p + 2 \times x$

$= p + 2x = p + 2(1 - 2p) = p + 2 - 4p = 2 - 3p$

Now, $\Sigma x_i^2 p_i = 0 + 1 \times p + 4 \times x$

$= p + 4x = p + 4(1 - 2p) = 4 - 7p$

$\because \quad \sum x_i^2 p_i = \sum x_i p_i$ [Given]

$\Rightarrow \quad 4 - 7p = 2 - 3p \Rightarrow 2 = 4p \Rightarrow \quad p = \frac{1}{2}$

12. (a) : We have, $y = \log_{10} x + \log_e x$

$\Rightarrow \quad y = \frac{\log x}{\log 10} + \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x \log 10} + \frac{1}{x} = \frac{\log_{10} e}{x} + \frac{1}{x}$

13. (c) : The given equation can be written as

$y = A\cos(x + C_3) - Be^x$ where $A = C_1 + C_2$ and $B = C_4 e^{C_5}$

So, there are three independent variables, (A, B, C_3).

Hence, the differential equation will be of order 3.

14. (d) : The probability distribution of X is

X	0	1	2	3	4	Otherwise
$P(X)$	0.1	k	$2k$	$2k$	k	0

(A) We know that, $\sum_{i=1}^{n} p_i = 1$

Therefore, $0.1 + k + 2k + 2k + k = 1$

i.e., $k = 0.15$

(B) P(you study for atleast two hours) $= P(X \geq 2)$
$= P(X = 2) + P(X = 3) + P(X = 4)$
$= 2k + 2k + k = 5k = 5 \times 0.15 = 0.75$

(C) P(you study exactly two hours) $= P(X = 2)$
$= 2k = 2 \times 0.15 = 0.3$

(D) P(you study atmost two hours) $= P(X \leq 2)$
$= P(X = 0) + P(X = 1) + P(X = 2)$
$= 0.1 + k + 2k = 0.1 + 3k = 0.1 + 3 \times 0.15 = 0.55$

15. (a) : $\int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

$= \int \frac{dx}{1+3\sin^2 x+3\cos^2 x+5\cos^2 x}$

$= \int \frac{dx}{4+5\cos^2 x} = \int \frac{\sec^2 x}{4\sec^2 x+5}dx$

$= \int \frac{\sec^2 x}{4(1+\tan^2 x)+5}dx = \int \frac{\sec^2 x}{4\tan^2 x+9}dx$

Let $2\tan x = t$

$\Rightarrow \quad 2\sec^2 x\, dx = dt \Rightarrow \sec^2 x\, dx = \frac{1}{2}dt$

$\therefore \quad \int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx = \frac{1}{2}\int \frac{dt}{t^2+(3)^2}$

$= \frac{1}{2}\times\frac{1}{3}\tan^{-1}\frac{t}{3}+C \quad = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

16. (c)

17. (a) : We have, $\int \frac{x^2-1}{x^4+3x^2+1}dx = \int \frac{x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x^2+\frac{1}{x^2}+3\right)}$

$= \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1} \quad = \int \frac{dz}{z^2+1}$, where $z = x+\frac{1}{x}$

$= \tan^{-1} z + c = \tan^{-1}\left(x+\frac{1}{x}\right)+c$

18. (d) : The given system of equations are
$(k + 1)x + 8y = 0$, $kx + (k + 3)y = 0$

Coefficient matrix, $A = \begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = (k + 1)(k + 3) - 8k$

$= k^2 + 4k + 3 - 8k = k^2 - 4k + 3 = (k - 1)(k - 3)$

For unique solution $|A| \neq 0$ *i.e.*, k must not be equal to 1 or 3.

19. (a) : Given differential equation is
$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{(e^y+1)\cos x}{e^y \sin x}$

$\Rightarrow \quad \int \frac{e^y}{e^y+1}dy = -\int \frac{\cos x}{\sin x}dx + \log c$

$\Rightarrow \quad \log|e^y + 1| = -\log|\sin x| + \log c$

$\Rightarrow \quad \log|(e^y + 1)\sin x| = \log c \Rightarrow (e^y + 1)\sin x = c$

20. (c) : For reflexive, $a\ p\ a$

$\Rightarrow \quad a^2 - 4a^2 + 3a^2 = 0$

Which is true $\forall\ a \in R$

$\therefore \quad p$ is reflexive.

For symmetric : $a\ p\ b$

$\Rightarrow \quad a^2 - 4ab + 3b^2 = 0$

This doesn't implies that $b^2 - 4ab + 3a^2 = 0$

So, p is not symmetric.

Similarly, p is not transitive

21. (a)

22. (c) : $\lim_{x\to 0}\frac{1}{x}[f(x)+f(2x)+f(3x)+\ldots+f(2015x)]$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 0}\frac{[f'(x)+f'(2x)\cdot 2+f'(3x)\cdot 3+\ldots+f'(2015x)\cdot 2015]}{1}$

$= f'(0) + 2f'(0) + 3f'(0) + \ldots + 2015f'(0)$

$= (1 + 2 + 3 + \ldots + 2015)\, f'(0) = \frac{2015\cdot(2016)}{2}\cdot 2$

$= 2015 \times 2016$

23. (b) : We have $\int_a^b x^3 dx = 0, \int_a^b x^2 dx = \frac{2}{3}$

$f_1 = x^3, f_2 = x^2$.

f_1 is odd and f_2 is even function.

$\therefore \quad$ If $a = -1$ and $b = 1$, then

$\int_{-1}^{1} x^3 dx = 0$ and $\int_{-1}^{1} x^2 dx = 2\int_0^1 x^2 dx = \frac{2}{3}$

24. (b) : We have, $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4$

Now, $|\text{adj (adj(adj } 2A))| = |2A|^{(n-1)^3} = |2A|^{2^3} = |2A|^8$

$= \left[2^3 |A|\right]^8 = 8^8 \cdot 4^8 = 32^8 = 2^8 \cdot 16^8 = 16^2 \cdot 16^8 = 16^n$

$\Rightarrow \quad n = 10$

25. (d)

26. (c) : Given, $d_1 = 8\hat{i} - 6\hat{j}$ and $d_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$$\therefore \quad \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix}$$

$= \hat{i}(72 - 0) - \hat{j}(-96 - 0) + \hat{k}(32 + 18) = 72\hat{i} + 96\hat{j} + 50\hat{k}$

$\therefore \quad$ Required area $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$= \frac{1}{2}\sqrt{(72)^2 + (96)^2 + (50)^2} = 65$ sq. units

27. (d) : For one-one, $y = f(x) = \frac{2x}{x-1}, x \neq 1$

Suppose $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{2x_1}{x_1 - 1} = \frac{2x_2}{x_2 - 1} \Rightarrow x_1 = x_2$

So, f is one-one function.

For Range, $y = \frac{2x}{x-1}$

$\Rightarrow \quad yx - y = 2x \Rightarrow \quad x = \frac{y}{y-2} \quad y \neq 2$

$\therefore \quad$ Range = Co-domain

So, f is also an onto function.

28. (c) : P(atleast one of A and B)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B) \quad [\because A, B$ are independent$]$

$= P(A) + P(B)\,[1 - P(A)] = [1 - P(A')] + P(B)\,P(A')$

$= 1 - P(A') + P(B)\,P(A') = 1 - P(A')\,[1 - P(B)]$

$= 1 - P(A')\,P(B')$

29. (c) : If a line makes the angle α, β, γ with x, y, z axis respectively, then

$l^2 + m^2 + n^2 = 1$ or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Here, $\alpha = \gamma = \theta$

$\therefore \quad 2\cos^2\theta + \cos^2\beta = 1 \Rightarrow 2\cos^2\theta = \sin^2\beta$

$\Rightarrow \quad 2\cos^2\theta = 3\sin^2\theta \quad [\because \sin^2\beta = 3\sin^2\theta]$

$\Rightarrow \quad 2\cos^2\theta = 3(1 - \cos^2\theta) \Rightarrow 5\cos^2\theta = 3 \Rightarrow \cos^2\theta = \frac{3}{5}$

30. (d) : We have, $A^2 = A$

Now, $(I + A)^3 = I^3 + A^3 + 3I^2A + 3IA^2$

$= I + A^2 \cdot A + 3A + 3A^2 = I + A \cdot A + 3A + 3A \quad (\because A^2 = A)$

$= I + A^2 + 6A = I + A + 6A = I + 7A$

31. (b) : Clearly, $-1 \leq [x] \leq 1$

$\Rightarrow -1 \leq x < 2 \Rightarrow x \in [-1, 2)$

32. (a) : The equation of the curve is given as

$y = 8x + \cos y$

On differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = 8\frac{dx}{dt} - \sin y \frac{dy}{dt} \qquad \ldots(i)$$

It is given that $4\frac{dx}{dt} = \frac{dy}{dt}$, then from (i), we get

$-\frac{dy}{dt} + \sin y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt}(\sin y - 1) = 0$

$\Rightarrow \quad \sin y = 1 \Rightarrow y = \pi/2 \ \therefore \ \frac{\pi}{2} = 8x \Rightarrow x = \frac{\pi}{16}$

So, required point is $\left(\frac{\pi}{16}, \frac{\pi}{2}\right)$.

33. (d) : Given, $f(x) = \cot^{-1}x + x$

$\Rightarrow \quad f'(x) = \frac{-1}{1+x^2} + 1 \Rightarrow f'(x) = \frac{x^2}{1+x^2} \geq 0, \ \forall\, x \in R$

$\therefore \quad f(x)$ is increasing in $(-\infty, \infty)$.

34. (b)

EXAM ALERT 2025

Exam	Date
JEE Main Session 2	2nd, 3rd, 4th, 7th and 8th April
MHT-CET (PCB)	9th April to 17th April (Except 10th & 14th April)
MHT-CET (PCM)	19th April to 27th April (Except 24th April)
NDA I	13th April
K CET (PC)	16th April
K CET (MB)	17th April
AMUEEE	20th April
VITEEE	21st- 27th April (Tentative)
KEAM	Between 22nd April and 30 April
WB JEE	27th April
SRM JEEE	Phase 1 : 22nd April to 27th April Phase 2 : 12th June to 17th June Phase 3 : 4th July to 5th July
TG EAPCET (Agriculture & Pharmacy)	29th April & 30th April
TG EAPCET (Engineering)	2nd May to 5th May
NEET	4th May
CUET UG	Between 08 May and 01 June (Tentative)
COMEDK	10th May
JEE Advanced	18th May
BITSAT Session-1	26th to 30th May
BITSAT Session-2	22nd to 26th June
NDA II	14th September

35. (c) : We have, $P(A)=\frac{1}{4}, P(B)=\frac{1}{5}$ and $P(A\cap B)=\frac{1}{8}$

$$P\left(\frac{A^C}{B^C}\right)=\frac{P(A^C\cap B^C)}{P(B^C)}=\frac{P((A\cup B)^C)}{P(B^C)}$$

$$=\frac{1-P(A\cup B)}{1-P(B)}=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$=\frac{1-\frac{1}{4}-\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}}=\frac{40-10-8+5}{40\times\left(\frac{4}{5}\right)}=\frac{27}{32}$$

36. (b) : We have, $f(x)=|x^2-1|, x\in R$

$$\Rightarrow\quad f(x)=\begin{cases}x^2-1, & x\le -1 \text{ or } x\ge 1\\ -x^2+1, & x\in(-1,1)\end{cases}$$

$$\Rightarrow\quad f'(x)=\begin{cases}2x, & x\in(-\infty,-1)\cup(1,\infty)\\ -2x, & x\in(-1,1)\end{cases}$$

$$\Rightarrow\quad f''(x)=\begin{cases}2, & x\in(-\infty,-1)\cup(1,\infty)\\ -2, & x\in(-1,1)\end{cases}$$

$\therefore$ $f(x)$ has local minima at $x=\pm1$ and has a local maxima at $x=0$

37. (a) : Here, $|\vec{a}|=|\vec{b}|$

$$\therefore\quad |\vec{a}-\vec{b}|=\sqrt{|\vec{a}|^2+|\vec{a}|^2-2|\vec{a}|^2\cos\theta}$$

$$=\sqrt{2|\vec{a}|^2(1-\cos\theta)}=\sqrt{2|\vec{a}|^2\cdot 2\sin^2\frac{\theta}{2}}=2|\vec{a}|\sin\frac{\theta}{2}$$

$$\Rightarrow\quad |\vec{a}-\vec{b}|=2|\vec{a}|\sin\frac{\theta}{2}\quad\Rightarrow\quad |\vec{a}|\sin\frac{\theta}{2}=\frac{|\vec{a}-\vec{b}|}{2}$$

38. (d) : Given, $f(x)=\sum_{n=0}^{\infty}\frac{x^n}{n!}(\log a)^n$

$$\Rightarrow\quad f(x)=1+\frac{x}{1!}(\log a)+\frac{x^2}{2!}(\log a)^2+....\infty$$

$$\Rightarrow\quad f(x)=e^{x\log a}=a^x$$

At $x=0$, $f(x)=1$, which is continuous and differentiable.

39. (c) : $\because\quad x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$

Put $t=\tan\theta$ in both the equations, we get

$$x=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta \qquad ...(i)$$

and $$y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta \qquad ...(ii)$$

Differentiating (i) and (ii), we get

$$\frac{dx}{d\theta}=-2\sin 2\theta \text{ and } \frac{dy}{d\theta}=2\cos 2\theta$$

Therefore, $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{\cos 2\theta}{\sin 2\theta}=-\frac{x}{y}$

40. (d) : Value of $Z=px+qy$ at (15, 15) is $15p+15q$ and that at (0, 20) is $20q$.

According to given condition, we must have

$$15p+15q=20q\ \Rightarrow\ 15p=5q\ \Rightarrow\ q=3p$$

41. (a) : $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is defined if $\left|\frac{1+x}{x}\right|\ge 1$

$$\Rightarrow\quad \frac{(1+x)^2}{x^2}\ge 1\Rightarrow 1+x^2+2x\ge x^2\Rightarrow x\ge\frac{-1}{2}$$

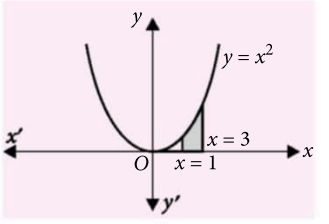
Also, for $x=0$, $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is undefined.

Domain of the function is $x\in\left[\frac{-1}{2},\infty\right)-\{0\}$.

42. (c) : (A) Required area

$$=\int_1^3 x^2\,dx=\left[\frac{x^3}{3}\right]_1^3$$

$$=\frac{1}{3}(27-1)=\frac{26}{3}\text{ sq. units}$$



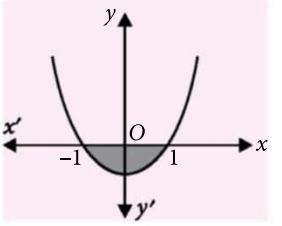
(B) The equation $y=x^2-1$ represents a parabola with vertex at (0, –1). It cuts x-axis where $y=0$.

$\therefore\quad x^2-1=0\Rightarrow x=\pm 1$

$\therefore$ Required area

$$=\left|\int_{-1}^{1}(x^2-1)\,dx\right|$$

$$=\left|\left[\frac{x^3}{3}\right]_{-1}^{1}-[x]_{-1}^{1}\right|$$



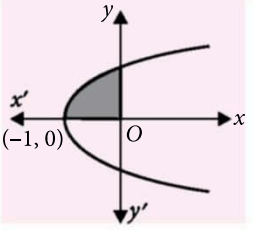
$$=\left|\frac{1}{3}(1+1)-(1+1)\right|=\left|\frac{2}{3}-2\right|=\frac{4}{3}\text{ sq. units}$$

(C) The equation $y^2=x+1$ represents a parabola with vertex at (–1, 0).

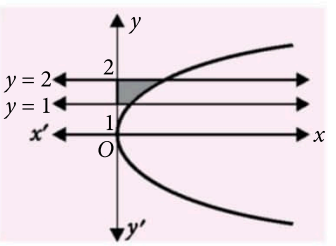
$\therefore$ Required area

$$=\int_{-1}^{0}\sqrt{x+1}\,dx=\frac{2}{3}\left[(x+1)^{3/2}\right]_{-1}^{0}$$

$$=\frac{2}{3}[(1)^{3/2}-0]=\frac{2}{3}\text{ sq. unit}$$



(D) The equation $y^2 = x$ represents a parabola with vertex at (0, 0).



$\therefore$ Required area

$$= \int_1^2 y^2\, dy$$

$$= \frac{1}{3}[y^3]_1^2 = \frac{1}{3}(8-1) = \frac{7}{3} \text{ sq. units}$$

43. (c) : (A) There are 30 days in April but each person can have birthday in 30 days.

$\therefore$ $n(S) = 30 \times 30 \times ... \times 30$ (10 times) $= 30^{10}$

$n(E) = n(S)$ – The number of ways in which 10 persons can have different birthdays.

$\therefore$ $n(E) = 30^{10} - {}^{30}C_{10}$

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{30^{10} - {}^{30}C_{10}}{30^{10}}$$

(C) is also true.

44. (b) : (A) $\cos^2 60° + \cos^2 45° + \cos^2 \alpha = 1$

$$\Rightarrow \quad \frac{1}{4} + \frac{1}{2} + \cos^2\alpha = 1 \ \Rightarrow\ 1 - \cos^2\alpha = \frac{3}{4}$$

$$\Rightarrow \quad \sin^2\alpha = \frac{3}{4}$$

(B) We have, angle between the lines whose direction ratios are 4, –3, 5 and 3, 4, 5 is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\cos\theta = \left|\frac{(4)(3) + (-3)(4) + (5)(5)}{\sqrt{16+9+25}\sqrt{9+16+25}}\right|$$

$$\Rightarrow \quad \cos\theta = \frac{25}{50} = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

(C) Given lines are $\frac{x}{-1} = \frac{y}{1} = \frac{z}{1}$

and $\frac{x-3}{0} = \frac{y+3}{1} = \frac{z-3}{-1}$

Distance between lines

$$= \left|\frac{\begin{vmatrix} 3-0 & -3-0 & 3-0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}}{\sqrt{(-1-1)^2 + (0-1)^2 + (-1-0)^2}}\right|$$

$$= \left|\frac{\begin{vmatrix} 3 & -3 & 3 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}}{\sqrt{(-2)^2 + (-1)^2 + (-1)^2}}\right|$$

$$= \frac{|3(-1-1) + 3(1-0) + 3(-1-0)|}{\sqrt{4+1+1}} = \frac{|-6|}{\sqrt{6}} = \sqrt{6} \text{ units}$$

45. (c)

46. (a) : $\frac{dy}{dx} + \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right)y = \frac{1}{x}$

I.F. $= \exp\int\left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right)dx$

$= \exp \ln\left(x \tan\frac{x}{2}\sin x\right) = x\tan\frac{x}{2} \times 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= x(1 - \cos x)$

$\therefore$ Solution is, $yx(1 - \cos x)$

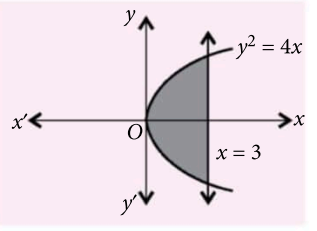
$$= \int \frac{1}{x} \cdot x(1-\cos x)\, dx = x - \sin x + c$$

$$y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \ \Rightarrow\ c = 0$$

$$\therefore\quad y(x) = \frac{x - \sin x}{x(1-\cos x)}$$

47. (a) : The given curve is $y^2 = 4x$, which represents a right hand parabola with vertex at (0, 0) and axis along x-axis and the equation of line is $x = 3$.

Required area = 2 × (area of shaded region in the first quadrant only)



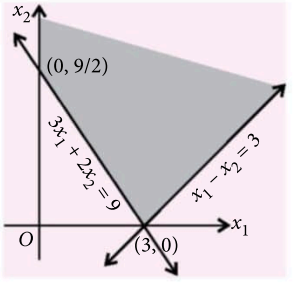
$$= 2\int_0^3 |y|\, dx = 2\int_0^3 2\sqrt{x}\, dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = \frac{8}{3}[3^{3/2} - 0]$$

$$= \frac{8}{3}(3\sqrt{3}) = 8\sqrt{3} \text{ sq. units}$$

48. (b) : The feasible region of the given constraints is shown shaded as in the figure.

Here, the shaded region shown in the figure is unbounded. Clearly, x_1 and x_2 can take arbitrary large values. So, the objective function can be made as large as possible. Therefore, the LPP has unbounded solution.



49. (a) : We have, $P(E)=\frac{13}{52}=\frac{1}{4}$ and $P(F)=\frac{4}{52}=\frac{1}{13}$

Also, $E \cap F$ denote the event that the card drawn is the ace of spades.

$\therefore\ P(E\cap F)=\frac{1}{52}$

Hence, $P(E\,|\,F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{52}}{\frac{1}{13}}=\frac{1}{4}$

Since, $P(E)=\frac{1}{4}=P(E\,|\,F)$, we can say that the occurrence of event F has not affected the probability of occurrence of the event E. We also have,

$$P(F\,|\,E)=\frac{P(E\cap F)}{P(E)}=\frac{\frac{1}{52}}{\frac{1}{4}}=\frac{1}{13}$$

50. (b)

51. (a)

52. (d) : We have, $A' = -A$ $(\because\ A$ is skew- symmetric)

Now, $(A^{2021})' = (A')^{2021} = (-A)^{2021} = -1(A^{2021})$

$\therefore$ A^{2021} is also skew symmetric matrix

53. (c) : Given, $y = \alpha x^2 - 6x + \beta$ passes through (0, 2)

$\therefore$ $\beta = 2$

Also, $\frac{dy}{dx}=2\alpha x-6$

Now, according to given condition, $\left(\frac{dy}{dx}\right)_{x=\frac{3}{2}}=0$

$\Rightarrow 2\alpha\left(\frac{3}{2}\right)-6=0 \ \Rightarrow\ 3\alpha=6 \Rightarrow\ \alpha=2$

54. (a) : Since, $3^2 = 9 = 1 \bmod 4$

Now, $3^{16} = (3^2)^8 = 1^8 \bmod 4 \Rightarrow\ 3^{16} \equiv 1\ (\text{mod } 4)$

$\therefore$ $3^{16}\ (\text{mod } 4) \equiv 1.$

55. (b) : Let $I=\int e^x\sqrt{e^x}\,dx$

$=\int e^x e^{x/2}dx=\int e^{3x/2}dx=\frac{2}{3}e^{3x/2}+C\ =\frac{2}{3}e^x\sqrt{e^x}+C$

56. (d) : Average Cost (AC) $=\frac{C(x)}{x}$

Marginal Cost $=\frac{d}{dx}C(x)$

Marginal Average Cost $=\frac{1}{x}(MC-AC)$

Marginal Revenue $=\frac{d}{dx}R(x)$

57. (c)

58. (b) : Given, $I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x}{1+5^x}dx$ ...(i)

Also, $I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2(\pi/4-\pi/4-x)}{1+5^{(\pi/4-\pi/4-x)}}dx$

$$\left\{\because\ \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right\}$$

$\Rightarrow\ I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x}{1+5^{-x}}dx\ \Rightarrow\ I=\int_{-\pi/4}^{\pi/4}\frac{5^x\tan^2 x}{5^x+1}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_{-\pi/4}^{\pi/4}\frac{\tan^2 x\,(1+5^x)}{1+5^x}dx\ =\int_{-\pi/4}^{\pi/4}\tan^2 x\,dx$$

59. (b) : Let the total profit be x.

$\because$ 10% of profit goes in charity,

$\therefore$ Profit shares by both the partner = 90% of $x=\frac{90x}{100}=\frac{9x}{10}$.

Now, Rakesh share of profit $=\frac{4}{9}\times\frac{9}{10}x\Rightarrow 18000=\frac{2}{5}x$

$\Rightarrow x = 45000$

Hence, the total profit is 45000.

60. (b) : (A) 21 mod (2) ≡ 1 mod (2)

$\therefore$ Remainder = 1

(B) (8 × 7) mod 6 ≡ 2 mod 6

So, remainder = 2.

(C) 12 mod 4 ≡ 0 mod 4

So, remainder = 0.

(D) (9 + 23)mod 12 = 8 mod 12

So, remainder = 8.

61. (c)

62. (b) : According to problem,

$y^2 = 4a(x - a)$ ...(i)

Differentiation w.r.t x, we get

$2y\frac{dy}{dx}=4a\ \Rightarrow\ \frac{dy}{dx}=\frac{2a}{y}\ \Rightarrow\ a=\frac{y}{2}\left(\frac{dy}{dx}\right)$

Putting value of a in (i), we get

$$y^2=4\frac{y}{2}\left(\frac{dy}{dx}\right)\left(x-\frac{y}{2}\left(\frac{dy}{dx}\right)\right)$$

$$\Rightarrow\ y^2=2yx\frac{dy}{dx}-y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow\ y=2x\frac{dy}{dx}-y\left(\frac{dy}{dx}\right)^2\ \Rightarrow\ y\left(\frac{dy}{dx}\right)^2=2x\frac{dy}{dx}-y$$

63. (b) : We have, $np = 8$ and $npq = 4$

$\Rightarrow \quad q = \frac{4}{8} = \frac{1}{2} \quad \therefore \quad p = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow n = 16$

Now, $P(X = r) = {}^{n}C_r\, p^r q^{n-r}$

$\therefore \quad P(X = 1) = {}^{16}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{15} = 16 \times \frac{1}{2^{16}} = \frac{2^4}{2^{16}} = \frac{1}{2^{12}}$

64. (b) : Given, $3P(X = 4) = \frac{1}{2}P(X = 2) + P(X = 0)$

$\therefore \quad P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad [x = 0, 1, 2, \ldots]$

Now, $3P(X = 4) = \frac{1}{2}P(X = 2) + P(X = 0)$

$\Rightarrow \quad \frac{3e^{-\lambda}\lambda^4}{4!} = \frac{1}{2}\frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^0}{0!} \Rightarrow \frac{3\lambda^4}{4!} = \frac{\lambda^2}{4} + 1$

$\Rightarrow \quad \frac{\lambda^4}{8} = \frac{\lambda^2}{4} + 1 \text{ or } \lambda^4 - 2\lambda^2 - 8 = 0$

$\Rightarrow \quad \lambda^2 = 4 \quad \Rightarrow \quad \lambda = 2 \quad [\because \quad \lambda^2 \neq -2]$

$\therefore \quad$ Mean of $X = 2$

65. (a)

66. (b) : The given differential equation can be written as $1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^3$

Clearly, degree of given differential equation is 3 and order is 2.

67. (c) : Market trend does not affect EMI.

68. (a) : Given, upstream speed $v = 10$ km/h
and downstream speed $u = 18$ km/h

$\therefore \quad$ Speed of boat in still water

$x = \frac{u+v}{2} = \frac{18+10}{2} = 14$ km/h and speed of stream

$y = \frac{u-v}{2}$ and $\frac{18-10}{2} = 4$ km/h

69. (d) : All of the given tests are the test of adequacy of index numbers.

70. (c) : Given, A is a skew-symmetric matrix.

$\therefore \quad A' = -A$

Also, $A^2 + I = 0 \Rightarrow A^2 = -I$

$\Rightarrow \quad A \cdot A = -I \Rightarrow A \cdot A \cdot A' = -I\,A' = I\,(-A') = IA = A$

$\Rightarrow \quad (A^{-1}A)\,AA' = A^{-1}A \Rightarrow AA' = I$

Do's and Dont's while Preparing for CUET

☑ Do's

✓ **Build a Strong Foundation**

- Understand concept clearly, mainly focus on definitions, theorems and formulas rather than rote learning.

✓ **Follow a Study Plan**

- Allocate time for each chapter based on its weightage. Start with easy and high scoring topics- like Relations and Functions, Differentiation, Integration and Probability.

✓ **Practice, Practice and Practice**

- Solve NCERT exercise first. Also, attempt previous year questions and sample paper.

✓ **Improve Problem Solving Skills**

- Analyse your error and correct them.

☒ Dont's

× **Don't Ignore Basics and Theorems**

- Skipping definitions and theorems can lead to conceptual confusion.

× **Don't Memorise without Understanding**

- Understand derivations and concepts instead of cramming formulas.

× **Don't Leave Calculus for the last Moment**

- Calculus has high weightage. Practice differentiation and integration regularly.

× **Don't Neglect Time Management**

- Always set a timer while solving papers to improve speed and accuracy.

× **Don't Panic Over Difficult Questions**

- If you stuck, move to the next question and come back later.

Regular Practice → Smart Strategy → High Marks

71. (b) : (A) Since $|x+3| \geq 10$

$\Rightarrow \quad x+3 \leq -10 \quad$ or $\quad x+3 \geq 10$

$\Rightarrow \quad x \leq -13$ or $x \geq 7$

$\Rightarrow \quad x \in (-\infty, -13] \cup [7, \infty)$

(B) We have, $\dfrac{x-7}{x+3} > 2$

or $\dfrac{x-7}{x+3} - 2 > 0$ or $\dfrac{x+13}{x+3} < 0$

$\Rightarrow$ $(x + 13 > 0$ and $x + 3 < 0)$

or $(x + 13 < 0$ and $x + 3 > 0)$

$\Rightarrow$ $(x > -13$ and $x < -3)$

or $(x < -13$ and $x > -3)$, which is not possible

$\Rightarrow \quad x \in (-13, -3)$

(C) We have, $\dfrac{x-2}{x+5} > 2$

$\Rightarrow \quad \dfrac{x-2}{x+5} - 2 > 0$

$\Rightarrow \quad \dfrac{-(x+12)}{x+5} > 0 \Rightarrow \quad \dfrac{x+12}{x+5} < 0$

$\Rightarrow$ $(x + 12 > 0$ and $x + 5 < 0)$ or $(x + 12 < 0$ and $x + 5 > 0)$

$\Rightarrow$ $(x > -12$ and $x < -5)$

or $(x < -12$ and $x > -5)$ (Not possible)

Therefore, $-12 < x < -5$, *i.e.*, $x \in (-12, -5)$

72. (b) : We have, $A^2 = A$ and $B = I - A$

Now, $AB + BA + I - (I - A)^2$

$= AB + BA + I - (I + A^2 - 2A)$

$= AB + BA - A + 2A \quad (\because A^2 = A)$

$= AB + BA + A = A(I - A) + (I - A)A + A$

$= A - A^2 + A - A^2 + A = A - A + A - A + A = A$

73. (c)

74. (c) : Seasonal variation

75. (a)

76. (a) : We have, $x = k(A - \sin A)$, $y = k(1 - \cos A)$

$$X'(A) = \frac{dx}{dA} = k(1 - \cos A)$$

$$Y'(A) = \frac{dy}{dA} = k \sin A$$

$$\frac{dy}{dx} = \frac{k \sin A}{k(1 - \cos A)} = \frac{2 \sin A/2 \cos A/2}{2 \sin^2 A/2} = \frac{\cos A/2}{\sin A/2} = \cot A/2$$

Now, $\dfrac{d^2y}{dx^2} = \dfrac{d}{dA}\left(\dfrac{dy}{dx}\right) \cdot \dfrac{dA}{dx}$

$$= \frac{1}{2}\left(-\operatorname{cosec}^2 \frac{A}{2}\right) \cdot \frac{1}{k(1 - \cos A)}$$

77. (c) : We have, $n = 10$

Probability of getting an odd number, $(p) = \dfrac{3}{6} = \dfrac{1}{2}$

$\therefore \quad q = 1 - p = \dfrac{1}{2}$

So, required probability = $P(x \geq 1)$

$$= 1 - P(x = 0) = 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 = 1 - \frac{1}{1024} = \frac{1023}{1024}$$

78. (d) : Variance is not a time series component, it refers to the spread of a data set.

79. (a) : (A) Here, $f(x) = \dfrac{x}{(1+x^2)} \Rightarrow f'(x) = \dfrac{(1-x^2)}{(1+x^2)^2}$

Given, $f(x)$ is decreasing function.

$\Rightarrow \quad f'(x) < 0 \quad \Rightarrow \quad \dfrac{(1-x^2)}{(1+x^2)^2} < 0$

$\Rightarrow \quad 1 - x^2 < 0 \quad \Rightarrow \quad x^2 - 1 > 0$

$\therefore \quad x \in (-\infty, -1) \cup (1, \infty)$

(B) Here, $f(x) = \tan^{-1} x - x$

$$\Rightarrow \quad f'(x) = \frac{1}{1+x^2} - 1 = -\frac{x^2}{1+x^2} < 0$$

($\because$ $f(x)$ is decreasing function)

$\therefore \quad f(x)$ decreases for all $x \in (-\infty, \infty)$.

(C) Here, $f(x) = x - e^x + \tan\left(\dfrac{2\pi}{7}\right)$

$\Rightarrow f'(x) = 1 - e^x > 0 \quad (\because f(x)$ is increasing function)

$\Rightarrow \quad e^x < 1 \Rightarrow e^x < e^0 \Rightarrow x < 0 \Rightarrow x \in (-\infty, 0)$

80. (a)

81. (c)

82. (b)

83. (c) : We find $(672 + 541 + 383 + 295 + 101 + 86) \bmod 3$

Since $672 \equiv 0 \pmod 3$, $541 \equiv 1 \pmod 3$, $383 \equiv 2 \pmod 3$, $295 \equiv 1 \pmod 3$, $101 \equiv 2 \pmod 3$ and $86 \equiv 2 \pmod 3$.

So, $(672 + 541 + 383 + 295 + 101 + 86) \bmod 3$

$\equiv (0 + 1 + 2 + 1 + 2 + 2) \bmod 3 \equiv 8 \pmod 3 \equiv$ $2 \pmod 3$.

Hence, the remainder when $(672 + 541 + 383 + 295 + 101 + 86)$ is divided by 3 is 2.

84. (c) : $f(x) = x^3 - 3x^2 + 3x + 4$; $f'(x) = 3x^2 - 6x + 3$.

For critical point, put $f'(x) = 0$ *i.e.*, $3x^2 - 6x + 3 = 0$

$\Rightarrow \quad 3(x^2 - 2x + 1) = 0 \Rightarrow (x - 1)^2 = 0$

$\Rightarrow \quad x = 1$ is the critical point

85. (b)

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. Let $f(x)$ and $g(x)$ be differentiable functions defined on [0,2] such that $f''(x) - g''(x) = 0$, $f'(1) = 2g'(1) = 4$, $f(2) = 3g(2) = 9$. Then at $x = \frac{3}{2}$, $f(x) - g(x)$ is

(a) 0 (b) 2 (c) 5 (d) 10

2. The area (in sq. units) of the portion of the circle $x^2 + y^2 = 64$ which is exterior to the curve $y^2 = 12x$ is

(a) $\frac{8}{3}\left(8\pi + \sqrt{3}\right)$ (b) $\frac{8}{3}\left(8\pi - \sqrt{3}\right)$

(c) $\frac{16}{3}\left(8\pi + \sqrt{3}\right)$ (d) $\frac{16}{3}\left(8\pi - \sqrt{3}\right)$

3. A line from the origin meets the lines

$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$

at P and Q respectively. If length $PQ = d$, then d^2 is

(a) 5 (b) 6 (c) 7 (d) 8

4. If s_1, s_2, s_3, ... are the sum of infinite geometric series whose first terms are 1, 2, 3, ... and whose common ratios are, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ respectively, then $s_1^2 + s_2^2 + s_3^2 + ... + s_{10}^2 =$

(a) 485 (b) 495 (c) 500 (d) 505

5. The maximum value of $f(x) = \int_0^2 \frac{dt}{2+|x-t|}$, $0 \le x \le 2$, is

(a) ln 2 (b) $\ln\frac{3}{2}$

(c) $2\ln\frac{3}{2}$ (d) ln 3

6. A man moves one unit distance for each step he takes. He always moves either parallel to the x-axis or y-axis. He starts at the point (0, 0) and reaches the point (1, 1) at the end of six steps. The number of ways he can do it is

(a) 280 (b) 300 (c) 360 (d) 420

7. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(a) g is not differentiable at $x = 0$

(b) $g'(0) = \cos(\log 2)$

(c) $g'(0) = -\cos(\log 2)$

(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

8. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c / G)$ equals

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$

(c) $P(E^c) - P(F)$ (d) $P(E) - P(F_c)$

9. A chord of the hyperbola $x^2 - 2y^2 = 1$ is bisected at the point (–1, 1). The area (in sq. units) bounded by the chord and the coordinate axes is

(a) 1 (b) 2 (c) 1/2 (d) 1/4

10. If $f(x)$, $g(x)$ are twice differentiable functions on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 2$, $g'(1) = 4$ and $f(2) = 3$, $g(2) = 9$, then $f(x) - g(x)$ at $x = 4$ equals

(a) 6 (b) –10 (c) 8 (d) 2

SECTION-II

Numerical Answer Type

11. If the function f satisfies the relation $f(x + y) = f(x) \cdot f(y)$ for all natural numbers x, y, $f(1) = 2$ and $\sum_{r=1}^{n} f(a+r) = 16(2^n - 1)$, then the natural number a is _____.

12. Let $f(x+y)=f(x)+f(y)-2xy-1$ for all x and y. If $f'(0)$ exists and $f'(0)=-\sin\alpha$, then the value of $f\{f'(0)\}$ is _____.

13. Let $a_r=r({}^7C_r)$, $b_r=(7-r)({}^7C_r)$ and $A_r=\begin{pmatrix} a_r & 0 \\ 0 & b_r \end{pmatrix}$. If $A=\sum_{r=0}^{7}A_r=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, then the value of $\frac{(a+b)}{128}$ must be _____.

14. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. The dot product of the vector in the same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$ with vector $3\hat{i}+\hat{j}$ is _____.

15. If the foot of perpendicular from the point $P(4, 6, 2)$ to the line $x-y-z+2=0=2x-y-4z+6$ is (a, b, c), then $\frac{a+b+c}{3}$ is _____.

SOLUTIONS

1. (c): Let $h(x)=f(x)-g(x)$ ...(i)

$h''(x)=f''(x)-g''(x)=0\Rightarrow h'(x)=a$

$h'(x)=f'(x)-g'(x)=a$

$h'(1)=f'(1)-g'(1)=a$, $4-2=a=2$

$h'(x)=2\Rightarrow h(x)=2x+b$

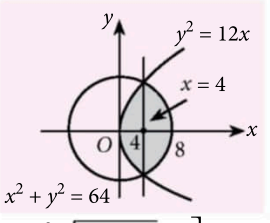
$h(2)=f(2)-g(2)=9-3=4+b\Rightarrow b=2$

$\Rightarrow h(x)=2x+2$ ...(ii)

$h\left(\frac{3}{2}\right)=f\left(\frac{3}{2}\right)-g\left(\frac{3}{2}\right)=5$ (From (i) & (ii))

2. (d): The curve meet at $x=4$.

The area of the shaded part



$=2\left[\int_0^4\sqrt{12x}\,dx+\int_4^8\sqrt{64-x^2}\,dx\right]$

$=\frac{64}{\sqrt{3}}+\left|x\sqrt{64-x^2}+64\sin^{-1}\frac{x}{8}\right|_4^8$

$=\frac{64}{\sqrt{3}}-16\sqrt{3}+\frac{64\pi}{3}=\frac{64\pi}{3}+\frac{16}{\sqrt{3}}$

$\therefore$ Required area $=64\pi-\left(\frac{64\pi}{3}+\frac{16}{\sqrt{3}}\right)=\frac{128\pi}{3}-\frac{16}{\sqrt{3}}$

$=\frac{16}{3}\left(8\pi-\sqrt{3}\right)$ sq. units

3. (b): $P=(2+s, 1-2s, -1+s)$

$Q=\left(\frac{8}{3}+2t, -3-t, 1+t\right)$

O, P, Q are collinear.

$\therefore \frac{2+s}{\frac{8}{3}+2t}=\frac{1-2s}{-3-t}=\frac{-1+s}{1+t}$

Solving, $s=3$, $t=\frac{1}{3}$

$\Rightarrow P=(5,-5,2)$, $Q=\left(\frac{10}{3},-\frac{10}{3},\frac{4}{3}\right)$

$d^2=(PQ)^2=\frac{25}{9}+\frac{25}{9}+\frac{4}{9}=6$

4. (d): s_r is the sum of an infinite G.P. with first term r and common ratio $\frac{1}{r+1}$.

$\therefore s_r=\frac{r}{1-\frac{1}{r+1}}=r+1$

$\Rightarrow \sum_{r=1}^{10}s_r^2=2^2+3^2+\ldots+11^2$

$=1^2+2^2+3^2+\ldots+11^2-1=\frac{11\times12\times23}{6}-1=505$

5. (c): $f(x)=\int_0^x\frac{dt}{2+x-t}+\int_x^2\frac{dt}{t+2-x}$

$=-\ln(2+x-t)\Big|_0^x+\ln(t+2-x)\Big|_x^2$

$=-\ln 2+\ln(2+x)+\ln(4-x)-\ln 2$

$=\ln\left(\frac{(2+x)(4-x)}{4}\right)$.

By symmetry, the maximum is $f(1)=\ln\left(\frac{9}{4}\right)=2\ln\frac{3}{2}$

6. (b): Let E, W, N, S stand for one unit movement along positive x-direction, negative x-direction, positive y-direction, negative y-direction respectively. The sequences of 6 steps are

$ENNNSS$ with $\frac{6!}{3!2!}=60$ ways,

$EEWNNS$ with $\frac{6!}{2!2!}=180$ ways

$EEEWWN$ with $\frac{6!}{3!2!}=60$ ways.

Required number of ways = 60 + 180 + 60 = 300

7. (b): As we are concerned about differentiability at '0' in the vicinity of $\sin x$.

$f(x) = \log 2 - \sin x$

$g(x) = f(f(x)) = \log 2 - \sin(\log 2 - \sin x)$

As g is sum of two differentiable functions, so g is differentiable. $g'(x) = \cos(\log 2 - \sin x)\cdot\cos x$

Then $g'(0) = \cos(\log 2)$.

8. (c): Let $e = P(E)$, $f = P(F)$, $g = P(G)$

$P(E^c \cap F^c \cap G) = P(E^c \cap F^c) - P(E^c \cap F^c \cap G^c)$

$= P(E^c)P(F^c) - 1 + P(E \cup F \cup G)$

$= (1 - e)(1 - f) - 1 + e + f + g - ef - fg - ge = g(1 - e - f)$

$$\therefore\ P(E^c \cap F^c \,|\, G) = \frac{P(E^c \cap F^c \cap G)}{P(G)}$$

$$= 1 - e - f = P(E^c) - P(F)$$

9. (d): The chord with midpoint $(-1, 1)$ is $S_1 = S_{11}$.

$\therefore\ -x - 2y = 1 - 2 = -1 \Rightarrow x + 2y = 1$

It meets the x-axis at $A(1, 0)$ and y-axis at $B\left(0, \frac{1}{2}\right)$

Area of $\Delta OAB = \frac{1}{2}\cdot 1\cdot\frac{1}{2} = \frac{1}{4}$ sq. units

10. (b): $f'(x) - g'(x) =$ constant $[\because\ f''(x) = g''(x)]$

$f'(1) - g'(1) = -2$ as $f'(1) = 2$ and $g'(x) = 4$

$\Rightarrow f'(x) - g'(x) = -2 \Rightarrow (f - g)(x) = -2x + k$

$f(2) - g(2) = k - 4 \Rightarrow 3 - 9 = k - 4 \Rightarrow k = -2$

$f(4) - g(4) = k - 8 = -10$

11. (3): $f(2) = f(1 + 1) = f(1) \times f(1) = 2^2$,

$f(3) = f(2 + 1) = f(2) \times f(1) = 2^2 \times 2 = 2^3\ \ \therefore\ f(n) = 2^n$

$$\Rightarrow 16(2^n - 1) = \sum_{r=1}^{n} f(a+r) = \sum_{r=1}^{n} 2^{a+r}$$

$$= 2^a(2 + 2^2 + ... + 2^n) = 2^a \cdot 2\left(\frac{2^n - 1}{2 - 1}\right)$$

$= 2^{a+1}(2^n - 1)$

$\therefore\ 2^{a+1} = 16 \Rightarrow a = 3$

12. (1): $$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\{f(x) + f(h) - 2xh - 1\} - f(x)}{h}$$

(Using the given relation)

$$= \lim_{h\to 0}(-2x) + \lim_{h\to 0}\left(\frac{f(h) - 1}{h}\right)$$

$$= \lim_{h\to 0}(-2x) + \lim_{h\to 0}\left(\frac{f(h) - f(0)}{h}\right)$$

[Putting $x = 0 = y$ in the given relation we find $f(0) = f(0) + f(0) + 0 - 1 \Rightarrow f(0) = 1$]

$\therefore\ f'(x) = -2x + f'(0)$

$\Rightarrow f(x) = -x^2 - \sin\alpha\cdot x + C,\ \ f(0) = -0 - 0 + C \Rightarrow C = 1$

$\therefore\ f(x) = -x^2 - \sin\alpha\cdot x + 1$

So, $f\{f'(0)\} = f(-\sin\alpha) = -\sin^2\alpha + \sin^2\alpha + 1$

$\therefore\ f\{f'(0)\} = 1$

13. (7): $$\sum_{r=0}^{7} A_r = \sum_{r=0}^{7}\begin{pmatrix} a_r & 0 \\ 0 & b_r \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$\therefore\ a = \sum_{r=0}^{7} a_r = \sum_{r=0}^{7} r\cdot({}^7C_r)$$

and $$b = \sum_{r=0}^{7} b_r = \sum_{r=0}^{7}(7 - r)\cdot({}^7C_r)$$

$$\Rightarrow a + b = 7\sum_{r=0}^{7}{}^7C_r = 7(2^7) = 7\times 128 \Rightarrow \frac{a+b}{128} = 7$$

14. (5): Let $\vec{c} = \alpha\hat{i} + \beta\hat{j}$

Given that $\vec{b} \perp \vec{c}\ \ \therefore\ \ \vec{b}\cdot\vec{c} = 0$

$\Rightarrow\ (4\hat{i} + 3\hat{j})\cdot(\alpha\hat{i} + \beta\hat{j}) = 0$

$$\Rightarrow\ 4\alpha + 3\beta = 0\ \Rightarrow\ \frac{\alpha}{3} = \frac{\beta}{-4} = \lambda$$

$\Rightarrow\ \alpha = 3\lambda,\ \beta = -4\lambda$ ...(i)

Now let $\vec{a} = x\hat{i} + y\hat{j}$ be the required vectors.

Projection of $\vec{a}$ along $\vec{c}$ gives $$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{4x + 3y}{\sqrt{4^2 + 3^2}} = 1$$

$\Rightarrow\ 4x + 3y = 5$ ...(ii)

Also projection of $\vec{a}$ along $\vec{c}$ gives

$$\frac{\vec{a}\cdot\vec{c}}{|\vec{c}|} = 2 \Rightarrow \frac{\alpha x + \beta y}{\sqrt{\alpha^2 + \beta^2}} = 2$$

$\Rightarrow\ 3\lambda x - 4\lambda y = 10\lambda \Rightarrow 3x - 4y = 10$ ...(iii)

Solving (ii) and (iii), we get $x = 2$, $y = -1$

Therefore, the required vector is $2\hat{i} - \hat{j}$.

Hence, required dot product $= 6 - 1 = 5$

15. (4): The *d.r's* of the line are given by

$l - m - n = 0,\ 2l - m - 4n = 0 \Rightarrow l : m : n = 3 : 2 : 1$

A point on the line with $z = 0$ is given by

$x - y + 2 = 0,\ 2x - y + 6 = 0 \Rightarrow (x, y, z) = (-4, -2, 0)$

The line is $$\frac{x + 4}{3} = \frac{y + 2}{2} = \frac{z}{1} = k$$

$Q(3k - 4, 2k - 2, k)$ is a point on the line.

The *d.r's* of PQ are $3k - 8, 2k - 8, k - 2$

PQ is perpendicular to the line.

$\therefore\ 3(3k - 8) + 2(2k - 8) + k - 2 = 0 \Rightarrow k = 3$

The foot is $Q(5, 4, 3)$

$$\frac{a + b + c}{3} = \frac{5 + 4 + 3}{3} = 4$$

Quiz CLUB

Unlock Your Knowledge!

1. If M is a 3×3 skew symmetric matrix, then find $\det(M)$.
2. Let $a_1, a_2, a_3, \ldots a_n, \ldots$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then find the sum of its first 17 terms.
3. If $f:[-2, 2] \to R$ is defined by

$$f(x) = \begin{cases} \dfrac{\sqrt{1+cx} - \sqrt{1-cx}}{x} & \text{for } -2 \le x < 0 \\ \dfrac{x+3}{x+1} & \text{for } 0 \le x \le 2 \end{cases}$$

is continuous on $[-2, 2]$, then find the value of c.

4. If $f(5) = 7$ and $f'(5) = 7$, then find $\lim\limits_{x \to 5} \dfrac{x\, f(5) - 5f(x)}{x - 5}$.
5. If $2\sin^{-1} x - \cos^{-1} x = \dfrac{\pi}{2}$, then find the value of x.
6. Let $A = \{1, 2, 3, 4, 5\}$. Find the domain of the relation from A to A by $R = \{(x, y) : y = 2x - 1\}$.
7. Let $f(x) = x^3$ and $g(x) = 3^x$. Find the values of a such that $g(f(a)) = f(g(a))$.
8. Evaluate : $\int \dfrac{x^2 + 1}{x^2 - 1} dx$
9. Find distance between the focii of the ellipse $x = 3\cos\theta$, $y = 4\sin\theta$.
10. If A and B are square matrices of the same order and $AB = 3I$, then find A^{-1}.
11. If $\sin\theta + \text{cosec}\theta = 2$, then find the value of $\sin^6\theta + \text{cosec}^6\theta$.
12. The diagonal of a square is changing at the rate of 0.5 cm/sec. Then find the rate of change of area, when the area is 400 cm^2.
13. If $P(S) = 0.3$, $P(T) = 0.4$. S and T are independent events, then find $P(S \mid T)$.
14. If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$. Then find (A, B).
15. Find the area (in sq. units) bounded by the curves $y = \sqrt{|x|}$ and $y = \pm x$.
16. Find the equation of a straight line, perpendicular to $3x - 4y = 6$ and forming a triangle of area 6 squares units with coordinate axes.
17. Find the degree of differential equation representing the family of curves $y^2 = a(ax + b)$ where a and b are arbitrary constants.
18. Find the number of ways of selecting atleast 4 candidates from 8 candidates.
19. Let $f: R \to R$ be a continuous function which satisfies $f(x) = \int\limits_0^x f(t)dt$. Then find the value of $f(\log_e 5)$.
20. If x_1, x_2, ..., x_n are n observations such that $\sum\limits_{i=1}^{n} x_i^2 = 400$ and $\sum\limits_{i=1}^{n} x_i = 80$, then find the least value of n.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

SOLUTIONS TO MARCH 2025 QUIZ CLUB

1. $10c^3$
2. $6 + 11i$
3. 7 units
4. $2\cos a$
5. 671
6. $\dfrac{8}{7}$
7. 6
8. 6
9. B
10. 4920
11. 6
12. $10\sqrt{3}$ cm^2/s.
13. 1
14. $\pi/2$
15. 8/3
16. $f(x) + c$
17. $\pi/3$
18. $\sqrt{2}$
19. 2401
20. 2/7

Are you ready for JEE 2025?

Series 2

Top Weightage Topics for JEE

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

ANALYTICAL GEOMETRY

TWO-DIMENSIONAL GEOMETRY

STRAIGHT LINES

General Equation

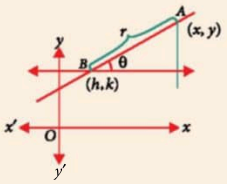
$ax + by + c = 0, |a| + |b| \neq 0$

Slope $= m = \tan\theta = -\frac{a}{b}$

x-intercept $= \frac{-c}{a}$, y-intercept $= \frac{-c}{b}$

Parametric Form

$\frac{x-h}{\cos\theta} = \frac{y-k}{\sin\theta} = r$



For $r > 0$; (x, y)

$= (h + r\cos\theta, k + r\sin\theta)$ and $\theta \in [0, \pi)$

For $r < 0$; $(x, y) = (h - r\cos\theta, k - r\sin\theta)$

Different Forms of Equation of Line

- **Slope-Intercept Form :** $y = mx + c$

  Here, m is the slope of the line making an intercept c on y-axis.

- **Point-Slope Form :** $\frac{y - y_1}{x - x_1} = m$

  Here, m is the slope and (x_1, y_1) be given point on the line.

- **Two Points Form :** $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) & (x_2, y_2) are two given points on the line

- **Intercept Form :** $\frac{x}{a} + \frac{y}{b} = 1$, where a and b represent x and y-intercepts respectively of the line.

- **Perpendicular Form or Normal Form :** $x\cos\alpha + y\sin\alpha = p$, where

  $\sin\alpha = \frac{b}{\sqrt{a^2 + b^2}}$, $\cos\alpha = \frac{a}{\sqrt{a^2 + b^2}}$, $p = \frac{-c}{\sqrt{a^2 + b^2}} > 0$

Position of two Points Relative to a given Line

Let the line be $L : ax + by + c = 0$ and two given points are $A(x_1, y_1)$ and $B(x_2, y_2)$. Then

- A and B are on the same side of L if
$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$
- A and B are on the opposite sides of L if
$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$$

Distance of a Point From a line

Distance of a point (x_1, y_1) from line $ax + by + c = 0$ is given by $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

Distance Between Two Parallel Lines

Distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Angle Between Two Lines

Let $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$ be two lines, then the acute angle (θ) between them is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|, \text{ where}$$

m_1 = slope of L_1, m_2 = slope of L_2

- L_1 and L_2 are perpendicular iff $m_1m_2 = -1$
- L_1 and L_2 are parallel iff $m_1 = m_2$

Equation of the Bisectors of Angle Between the Lines

Let $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ be two lines, then equation of angle bisectors are given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

CIRCLE

General Equation

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre = $(-g, -f)$

Radius = $\sqrt{g^2 + f^2 - c}$

Standard Equation

$(x - h)^2 + (y - k)^2 = r^2$, where r = radius of circle

Parametric Equation

$x = h + a\cos\theta$,
$y = k + a\sin\theta$,
where centre = (h, k), radius = a

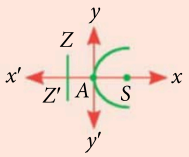
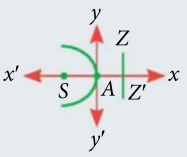
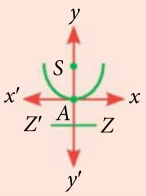
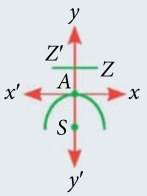
Diametric Equation

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, where (x_1, y_1) and (x_2, y_2) are end points of diameter.

Different Forms of Equations of Tangent and Normal

	Condition	Equation of Tangent	Equation of Normal
Slope Form	When a line of slope m and a circle $x^2 + y^2 = r^2$ is given	$y = mx \pm r\sqrt{1 + m^2}$	$my = -x \pm r\sqrt{1 + m^2}$
Point Form	When $P(x_1, y_1)$ is a point on the circle $x^2 + y^2 = r^2$	$xx_1 + yy_1 = r^2$	$xy_1 - yx_1 = 0$
	When $P(x_1, y_1)$ is a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$	$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$	$\frac{y - y_1}{y_1 + f} = \frac{x - x_1}{x_1 + g}$

PARABOLA

Standard Form	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Graph				
Vertex	$A(0, 0)$	$A(0, 0)$	$A(0, 0)$	$A(0, 0)$
Focus	$S(a, 0)$	$S(-a, 0)$	$S(0, a)$	$S(0, -a)$
Equation of Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of Directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Extremities of Latus Rectum	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$
Length of Latus Rectum	$4a$	$4a$	$4a$	$4a$

Different Forms of Equation of Tangent

Let the parabola be $y^2 = 4ax$

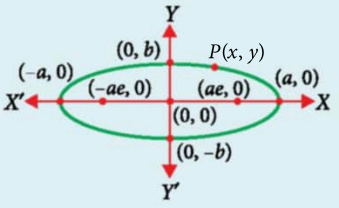
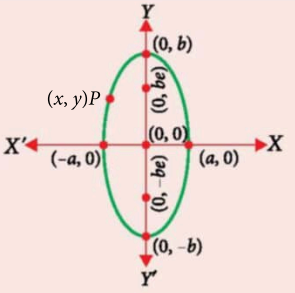
- **Point Form :** $yy_1 = 2a(x + x_1)$, where point is (x_1, y_1)
- **Parametric Form :** $yt = x + at^2$, where point is $(at^2, 2at)$
- **Slope Form :** $y = mx + \frac{a}{m}$, where point is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Different Forms of Equation of Normal

Let the parabola be $y^2 = 4ax$

- **Point Form** : $y - y_1 = \frac{-y_1}{2a}(x - x_1)$, where point is (x_1, y_1).
- **Parametric Form :** $y + tx = at^3 + 2at$, where point is $(at^2, 2at)$
- **Slope Form** : $y = mx - 2am - am^3$, where point is $(am^2, -2am)$

ELLIPSE

Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a < b$
Graph		

Centre	(0, 0)	(0, 0)
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of Major Axis	$2a$	$2b$
Length of Minor Axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Eccentricity	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{a^2}{b^2}}$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Different Forms of Equation of Tangent

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

- **Point Form :** $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$,

  where point is (x_1, y_1)

- **Parametric Form :** $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$,

  where $x = a\cos\theta, y = b\sin\theta$

- **Slope Form :** $y = mx \pm \sqrt{a^2m^2 + b^2}$

Different Forms of Equation of Normal

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

- **Point Form :** $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$,

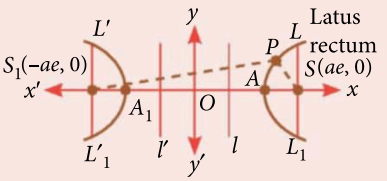
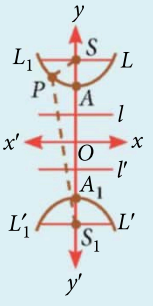
  where point is (x_1, y_1)

- **Parametric Form :** $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$,

  where $x = a\cos\theta, y = b\sin\theta$

- **Slope Form :** $y = mx \pm \frac{m(a^2 - b^2)}{\sqrt{a^2 + b^2m^2}}$

HYPERBOLA

Standard Form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Graph		

Centre	$(0, 0)$	$(0, 0)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$e = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \sqrt{1 + \frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latus rectum	$\left(\pm ae, \pm \frac{b^2}{a}\right)$	$\left(\pm \frac{a^2}{b}, \pm ae\right)$
Difference of focal radii $= \lvert SP - S'P \rvert$	$2a$	$2b$
Distance between foci	$2ae$	$2be$

Different Forms of the Equation of Tangent

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

- **Point Form :** $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$, where point is (x_1, y_1)
- **Parametric Form :** $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$, where $x = a \sec \theta$, $y = b \tan \theta$
- **Slope Form :** $y = mx \pm \sqrt{a^2m^2 - b^2}$

Different Forms of the Equation of Normal

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

- **Point Form :** $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$, where point is (x_1, y_1)
- **Parametric Form :** $ax\cos\theta + by\cot\theta = a^2 + b^2$
- **Slope Form :** $y = mx \mp \frac{m(a^2 + b^2)}{\sqrt{a^2 - b^2m^2}}$

THREE DIMENSIONAL GEOMETRY

Equation of a Line in Space

Vector form : Equation of a line passing through a point with position vector $\vec{a}$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$, $\lambda \in R$.

Cartesian form : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$, where (x_1, y_1, z_1) is passing point and a, b, c are direction ratios.

Equation of a Line Passing Through Two Given Points

Vector Form : A line passing through two given points having position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, where λ is a parameter.

Cartesian Form : Equation of a straight line joining two fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Angle Between Two Lines

Vector Form : If $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$, then

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

Cartesian Form : If $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$, then

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Conditions for Perpendicular and Parallel Lines :

- If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then lines are perpendicular.
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the lines are parallel.

Shortest Distance Between Two Lines

- **Vector Form :** $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$

  Then, shortest distance between them is given by

  $$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

- **Cartesian Form :** Let two skew lines be

  $l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and

  $l_2 : \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$, then the shortest distance between l_1 and l_2 is given by

  $$d = \left|\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - a_1c_2)^2 + (a_1b_2 - a_2b_1)^2}}\right|.$$

- If lines are parallel, then $d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$

Coplanarity of Two Lines

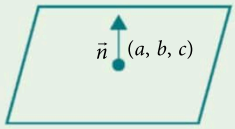
Vector Form : Two lines $\vec{r} = a_1 + \lambda_1\vec{b}_1$, $\vec{r} = \vec{a}_2 + \lambda_2\vec{b}_2$ are coplanar if and only if $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

Cartesian Form : The lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if and only if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$$

Plane

- Equation of Plane : $ax + by + cz + d = 0$,



$a^2 + b^2 + c^2 \neq 0$.

a, b, c are $d{\cdot}r$'s of the normal $\vec{n}$

- Equation of plane through the origin :

$ax + by + cz = 0$.

- Plane parallel to coordinate planes :

$x = k$, parallel to YOZ plane.

$y = k$, parallel to ZOX plane.

$z = k$, parallel to XOY plane.

- Plane parallel to coordinate axes :

$by + cz + d = 0$ is parallel to x-axis.

$ax + cz + d = 0$ is parallel to y-axis.

$ax + by + d = 0$ is parallel to z-axis.

- Intercept form : $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

It meets the axes at $A(a, 0, 0)$,

$B(0, b, 0)$,

$C(0, 0, c)$.

- Normal form : $lx + my + nz = p$,

p is the distance of the origin from the plane.

l, m, n are $d{\cdot}c$'s of the normal.

Vector form $\vec{r} \cdot \vec{n} = p$,

where $\vec{r} = x\breve{i} + y\breve{j} + z\breve{k}$, $\vec{n} = l\breve{i} + m\breve{j} + n\breve{k}$, unit normal vector.

- Plane through the point (x_1, y_1, z_1) :

$a(x - x_1) + b(y - y_1) + c(z - z_1)$

$= 0$.

- Plane through (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) :

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

- Coplanar points : (x_i, y_i, z_i), $i = 1, 2, 3, 4$ are coplanar.

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0.$$

- Distance of a point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

- Foot of perpendicular from a point (x_1, y_1, z_1) on the plane $ax + by + cz + d = 0$ is (x, y, z)

where $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

$= -\frac{(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$.

- Image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is (x, y, z), where

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

$= \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$.

A Line and A Plane

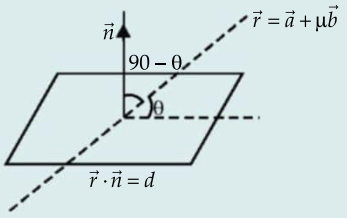
Let the line be $\frac{x-\alpha}{p}=\frac{y-\beta}{q}=\frac{z-\gamma}{r}$ and the plane be $ax+by+cz+d=0$ then,

(i) Line is normal to the plane : $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}$

(ii) Line is parallel to the plane : $ap+bq+cr=0$.

(iii) Line lies is the plane : $a\alpha+b\beta+c\gamma+d=0$, $ap+bq+cr=0$.

Angle between line and plane : Let line is $\vec{r}=\vec{a}+\mu\vec{b}$ and equation of plane is $\vec{r}\cdot\vec{n}=d$



$\therefore \sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$, where θ is the angle between the line and the plane.

- A line will be perpendicular to the plane (means it will be parallel to the normal to the plane)

  $\therefore \vec{b}\,||\,\lambda\vec{n}$ or $\vec{b}\times\vec{n}=0, \lambda\in R$

- A line will be parallel to the plane, so normal to the plane must be perpendicular to the line, then $\vec{b}\cdot\vec{n}=0$

- For cartesian form: If equation of line be $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and plane be given by $ax+by+cz+d=0$, then

  $\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$

  (where (l, m, n) are dc's of the line & (a, b, c) are dr's of normal to the plane $ax+by+cz+d=0$)

Two Planes

- Two planes $a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$ are said to be

(i) coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$,

(ii) parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\neq\frac{d_1}{d_2}$

(iii) perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$.

- Angle between the planes :

$$\theta=\cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}\right).$$

- Distance between parallel planes :

  $ax+by+cz+d_1=0$ and

  $ax+by+cz+d_2=0$ is $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$.

- Plane through the line of intersection of the planes

  $a_1x+b_1y+c_1z+d_1=0$,

  $a_2x+b_2y+c_2z+d_2=0$ is

  $a_1x+b_1y+c_1z+d_1+\lambda(a_2x+b_2y+c_2z+d_2)=0$.

Class 12

CBSE

SOLVED PAPER 2025

Held on 8th March

Hurray!! We are happy to inform our readers that in CBSE 2025 Mathematics question papers more than 70% questions were either exactly same or of similar type from **MTG Books**.

The references of few questions of paper having code : 65/1/2 are given here :

Paper Q. No.	P. No.	Q. No.	MTG Book
1	313	25	CBSE Champion
6	59	34	CBSE Champion
13	36	1	CBSE Champion
16	63	10	CBSE Champion
17	291	6	CBSE Champion
20	89	5	CBSE Champion
25	241	37	CBSE Champion
29 (a)	317	75	CBSE Champion

Paper Q. No.	P. No.	Q. No.	MTG Book
9	80	4	100 percent
20	163	3	100 percent
21 (b)	188	24	100 percent
30 (a)	351	39	100 percent
31	372	21	100 percent
32 (a)	210	19	100 percent
32 (b)	203	17	100 percent

and many more .....

General Instructions :

Time Allowed : 3 hours
Maximum Marks : 80

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. **All** questions are **compulsory**.
(ii) This question paper is divided into **FIVE** Sections – **A, B, C, D** and **E**.
(iii) In **Section A**–Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
(iv) In **Section B**–Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
(v) In **Section C**–Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
(vi) In **Section D**–Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
(vii) In **Section E**–Question Number **36** to **38** are case study based questions, carrying **4** marks each.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section–**B**, **3** questions in Section–**C**, **2** questions in Section–**D** and **2** questions in Section–**E**.
(ix) Use of calculator is **NOT** allowed.

SECTION - A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 marks each. **20 × 1 = 20**

1. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to :

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{7}{9}$

2. If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to :

(a) -18 (b) 18

(c) $\frac{-34}{9}$ (d) $\frac{34}{9}$

3. $\int \frac{1-2\sin x}{\cos^2 x}\,dx$ is equal to :

(a) $\tan x - 2\sec x + C$ (b) $-\tan x + 2\sec x + C$

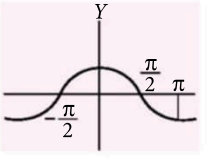
(c) $-\tan x - 2\sec x + C$ (d) $\tan x + 2\sec x + C$

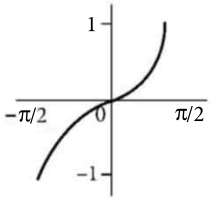
4. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = \sqrt{37}, |\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

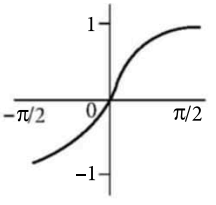
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

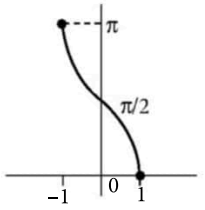
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

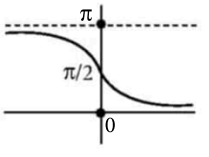
5. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



(a) 

(b) 

(c) 

(d) 

6. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9A^{-1})$ is equal to

(a) 9 (b) 9^2 (c) 9^3 (d) 9^4

7. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(a) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(b) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

(c) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

(d) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

8. Which of the following is not a homogeneous function of x and y?

(a) $y^2 - xy$ (b) $x - 3y$

(c) $\sin^2\frac{y}{x} + \frac{y}{x}$ (d) $\tan x - \sec y$

9. Let A be a matrix of order $m \times n$ and B is a matrix such that $A^T B$ and BA^T are defined. Then, the order of B is :

(a) $m \times m$ (b) $n \times n$

(c) $m \times n$ (d) $n \times m$

10. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?

(a) It will only have a maximum value.

(b) It will only have a minimum value.

(c) It will have both maximum and minimum values.

(d) It will have neither maximum nor minimum value.

11. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ is

(a) xe^x (b) $\frac{e^x}{x}$ (c) $\frac{x}{e^x}$ (d) $xe^{\frac{1}{x}}$

13. Let $A=[a_{ij}]$ be a square matrix of order 3 such that $a_{ij}=\hat{j}-2\hat{i}$. Then which of the following is true?

(a) $a_{12}>0$ (b) all $a_{ij}<0$

(c) $a_{13}+a_{31}=-6$ (d) $a_{23}>a_{32}$

14. The absolute maximum value of function $f(x)=x^3-3x+2$ in $[0, 2]$ is :

(a) 0 (b) 2 (c) 4 (d) 5

15. If $\int\frac{2^{\frac{1}{x}}}{x^2}dx=k\cdot 2^{\frac{1}{x}}+C$, then k is equal to

(a) $\frac{-1}{\log 2}$ (b) $-\log 2$ (c) -1 (d) $\frac{1}{2}$

16. If A and B are invertible matrices, then which of the following is not correct?

(a) $(A+B)^{-1}=B^{-1}+A^{-1}$ (b) $(AB)^{-1}=B^{-1}A^{-1}$

(c) $\text{adj}(A)=|A|A^{-1}$ (d) $|A|^{-1}=|A^{-1}|$

17. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z=18x+9y$ be the objective function, then

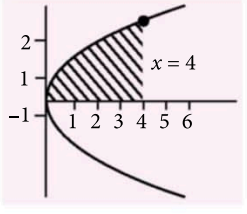
(a) Z is maximum at (2, 72), minimum at (15, 20)

(b) Z is maximum at (15, 20), minimum at (40, 15)

(c) Z is maximum at (40, 15), minimum at (15, 20)

(d) Z is maximum at (40, 15), minimum at (2, 72)

18. The area of the shaded region bounded by the curves $y^2=x$, $x=4$ and the x-axis is given by



(a) $\int_0^4 xdx$ (b) $\int_0^2 y^2dy$

(c) $2\int_0^4\sqrt{x}dx$ (d) $\int_0^4\sqrt{x}dx$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A)** : Let Z be the set of integers. A function $f: Z\to Z$ defined as $f(x)=3x-5$, $\forall\, x\in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

20. **Assertion (A)** : $f(x)=\begin{cases}3x-8, & x\le 5\\ 2k, & x>5\end{cases}$ is continuous at $x=5$ for $k=\frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x=a$,

$\lim\limits_{x\to a^-}f(x)=\lim\limits_{x\to a^+}f(x)=f(a)$.

SECTION - B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each. $5\times 2=10$

21. **(a)** Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$.

Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A (2, 1, 3) and B(8, −1, 0) respectively.

22. Find the values of 'a' for which $f(x)=x^2-2ax+b$ is an increasing function for $x>0$.

23. **(a)** Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2+y^2)=a^2$, then find $\frac{dy}{dx}$.

24. Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

25. The diagonals of a parallelogram are given by $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$. Find the area of the parallelogram.

SECTION - C

The section comprises of 6 Short Answer (SA) type questions of 3 marks each. $6 \times 3 = 18$

26. (a) Verify that lines given by $\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$ and $\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}-(2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B}=2\hat{i}+8\hat{j}$, $\vec{W}=6\hat{i}+12\hat{j}$ and $\vec{F}=12\hat{i}+18\hat{j}$, respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

27. Solve the following linear programming problem graphically :

Maximise $Z = 20x + 30y$

Subject to the constraints :

$x + y \le 80$

$2x + 3y \ge 100$

$x \ge 14$

$y \ge 14$

28. The area of an expanding rectangle is increasing at the rate of 48 cm^2/s. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth = 4.5 cm?

29. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
$P(X)$	p	$2p$	$3p$	p

Where X is the number of students absent.

(i) Calculate p.

(ii) Calculate the mean of the number of absent students on Saturday.

OR

(b) For the vacancy advertised in the newspaper. 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

30. (a) Find : $\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{2}}\frac{5\sin x+3\cos x}{\sin x+\cos x}dx$

31. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

SECTION - D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each. $4 \times 5 = 20$

32. (a) Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ w.r.t.

$\cos^{-1}\left(2x\sqrt{1+x^2}\right)$, $x\in\left(\frac{1}{\sqrt{2}},1\right)$

OR

(b) Find $\frac{dy}{dx}$, if $y = x^{\tan x}+\frac{\sqrt{x^2+1}}{2}$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

34. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions:

- The number of students in Sports clubs should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

35. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

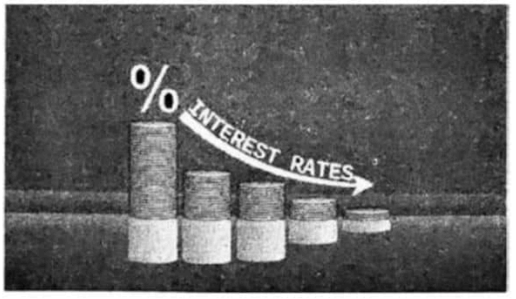
OR

(b) Find a point P on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q.

SECTION - E

This section comprises of 3 case study/passage based questions of 4 marks each. $3 \times 4 = 12$

36.



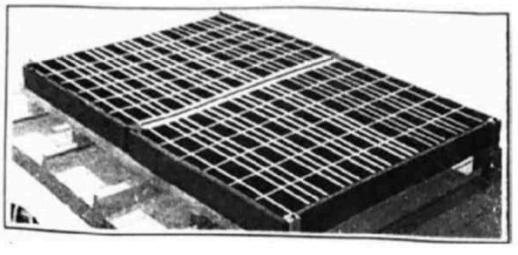
A bank offers loan to its customers on different type of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following:

(i) What is the probability that a customer after availing the loan will default on the loan repayment?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

37.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions:

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

38. A class-room teacher is keen to assess the learning of her study concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations :

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation?

SOLUTIONS

1. (c) : We have, $P(E)=\frac{2}{3}$ and $P(F)=\frac{3}{7}$

$P(E/\bar{F})=\frac{P(E\cap\bar{F})}{P(\bar{F})}$

$P(\bar{F})=1-P(F)=1-\frac{3}{7}=\frac{4}{7}$

So, $P(E/\bar{F})=\frac{\left(\frac{2}{3}\right)\times\left(\frac{4}{7}\right)}{\frac{4}{7}}=\frac{2}{3}$

[$\because$ E and F are independent events]

2. (b) : We have, $\vec{\alpha}=\hat{i}-4\hat{j}+9\hat{k}$ and $\vec{\beta}=2\hat{i}-8\hat{j}+\lambda\hat{k}$ are mutually parallel, then for $\vec{\alpha}$ and $\vec{\beta}$ to be parallel, $\vec{\beta}=p\vec{\alpha}$

$\Rightarrow 2\hat{i}-8\hat{j}+\lambda\hat{k}=p(\hat{i}-4\hat{j}+9\hat{k})$

$\Rightarrow 2\hat{i}-8\hat{j}+\lambda\hat{k}=p\hat{i}-4p\hat{j}+9p\hat{k}$

$\Rightarrow$ $p=2$ and $\lambda=9p$

So, $\lambda=9(2)$ or $\lambda=18$

3. (a) : We have, $\int\frac{1-2\sin x}{\cos^2 x}dx$

$=\int\frac{1}{\cos^2 x}dx-\int\frac{2\sin x}{\cos^2 x}dx$

$=\int\sec^2 dx-2\int\sec x\tan x\,dx$

$=\tan x-2\sec x+C$

4. (c) : We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$|\vec{a}|=\sqrt{37}, |\vec{b}|=3$ and $|\vec{c}|=4$

Now, $\vec{b}+\vec{c}=-\vec{a}$

Squaring both sides, $|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}=|\vec{a}|^2$

$\Rightarrow (3)^2+(4)^2+2(12)\cos\theta=(\sqrt{37})^2$

$\Rightarrow 9+16+24\cos\theta=37$

$\Rightarrow 24\cos\theta=12 \Rightarrow \cos\theta=\frac{1}{2}$ or $\theta=\frac{\pi}{3}$

$\therefore$ The angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

5. (c)

6. (b) : We have, A is a square matrix of order 3 and $|A|=9$.

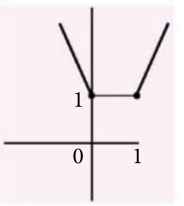
$\therefore |9A^{-1}|=9^3|A^{-1}|=9^3\left|\frac{1}{A}\right|$

$=9^3\times\frac{1}{9}=9^2$

7. (c) : We have, $f(x)=|x|+|x-1|$

Now, $|x|=\begin{cases}x, \text{if } x\geq 0\\ -x, \text{if } x<0\end{cases}$

and $|x-1|=\begin{cases}x-1, \text{if } x\geq 1\\ -x+1, \text{if } x<1\end{cases}$



$\therefore\ f(x)=|x|+|x-1|=\begin{cases}-2x+1, & \text{if } x<0\\ 1, & \text{if } 0\leq x<1\\ 2x-1, & \text{if } x\geq 1\end{cases}$

Clearly, from the graph, the function is continuous but not differentiable at $x=0$ and $x=1$.

8. (d)

9. (c) : We have, A is a matrix of order $m\times n$ and A^TB and BA^T are defined, then order of B is $m\times n$.

10. (c)

11. (d) : We have, $A=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$|A|=-1$

$A^{-1}=\begin{bmatrix}\frac{1}{-1}&0&0\\0&\frac{-1}{-1}&0\\0&0&\frac{-1}{-1}\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

12. (b) : We have, $\frac{dy}{dx}+y=\frac{1+y}{x} \Rightarrow \frac{dy}{dx}+y-\frac{y}{x}=\frac{1}{x}$

$\Rightarrow \frac{dy}{dx}+y\left(1-\frac{1}{x}\right)=\frac{1}{x}$

$\Rightarrow \frac{dy}{dx}+y\left(1-\frac{1}{x}\right)=\frac{1}{x},$

where $P=1-\frac{1}{x}$ and $Q=\frac{1}{x}$

$\therefore$ Integrating factor, $\text{I.F}=e^{\int Pdx}=e^{\int\left(1-\frac{1}{x}\right)dx}$

$=e^{[x-\log x]}=e^x e^{\log x^{-1}}=\frac{e^x}{x}$

13. (d) : We have, A is a square matrix of order 3 and $a_{ij}=\hat{j}-2\hat{i}$

Then, the matrix is $\begin{bmatrix}-1&0&1\\-3&-2&-1\\-5&-4&-3\end{bmatrix}$.

$\therefore$ The condition $a_{23}>a_{32}$ is correct.

14. (c) : We have, $f(x) = x^3 - 3x + 2$ in $[0, 2]$

Then, $f'(x) = 3x^2 - 3$. So, $f'(x) = 0$

$\Rightarrow \quad 3x^2 - 3 = 0 \quad \Rightarrow \quad x^2 = \frac{3}{3} = 1$ or $x = \pm 1$

$f(0) = 2, f(1) = 0, f(2) = 4$

$\therefore$ The maximum value is 4.

15. (a) : Let $I = \int \frac{2^{\frac{1}{x}}}{x^2} dx$

Putting $\frac{1}{x} = t \Rightarrow \frac{-1}{x^2} dx = dt$, we get

$\therefore \; I = \int -2^t \, dt = \frac{-2^t}{\log 2} + C = \frac{-2^{1/x}}{\log 2} + C$

$\therefore \quad k = \frac{-1}{\log 2}$

16. (a)

17. (c) :

Corner points	Value of $Z = 18x + 9y$
(2, 72)	18(2) + 9(72) = 684
(15, 20)	18(15) + 9(20) = 450 (Minimum)
(40, 15)	18(40) + 9(15) = 855 (Maximum)

18. (d)

19. (d) : Here, $f : Z \to Z$ as $f(x) = 3x - 5$

Let $a, b \in Z$

Taking $f(a) = f(b)$

$\Rightarrow \quad 3a - 5 = 3b - 5 \quad \Rightarrow \quad 3a = 3b \Rightarrow a = b$

$\therefore \quad f(x)$ is an injective.

For $y \in Z$, $y = 3x - 5$

$\Rightarrow \quad 3x = y + 5$

$\Rightarrow \quad x = \frac{y+5}{3} \notin Z$ For $\forall \; y \in Z$

$\therefore \quad f(x)$ is not a surjective.

Hence, $f(x)$ is not bijective.

$\therefore \quad$ Assertion is false

Clearly, Reason is true.

20. (d) : Given, f is continuous at $x = 5$.

$\lim\limits_{x \to 5^-} f(x) = \lim\limits_{x \to 5^+} f(x)$

$\Rightarrow \quad \lim\limits_{x \to 5^-} (3x - 8) = 2k \;\Rightarrow\; 7 = 2k \Rightarrow k = \frac{7}{2}$

$\therefore \quad$ Assertion is false and clearly, reason is true.

21. (a) We have, $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

$\therefore \; \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{(3\hat{i} + \hat{j} + 2\hat{k})(2\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}}$

$= \frac{6 - 2 + 8}{\sqrt{14}\sqrt{24}} = \frac{12}{\sqrt{14}\, 2\sqrt{6}} = \sqrt{\frac{6}{14}} = \sqrt{\frac{3}{7}}$

$\Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{3}{7}}$

OR

(b) $\overrightarrow{AB} = (8-2)\hat{i} + (-1-1)\hat{j} + (0-3)\hat{k} = 6\hat{i} - 2\hat{j} - 3\hat{k}$

$|\overrightarrow{AB}| = \sqrt{36+4+9} = \sqrt{49} = 7$

$\therefore \quad$ Unit vector direction opposite to $\overrightarrow{AB}$

$= \frac{-(6\hat{i} - 2\hat{j} - 3\hat{k})}{7} = \frac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{7}$

$\therefore \quad$ Required vector $= 21 \times \frac{(-6\hat{i} + 2\hat{j} + 3\hat{k})}{7}$

$= -18\hat{i} + 6\hat{j} + 9\hat{k}$

22. Given, $f(x) = x^2 - 2ax + b$; $f'(x) = 2x - 2a$

Given, f is an increasing function

$\therefore \quad f'(x) \geq 0$

$\Rightarrow \quad 2x - 2a \geq 0 \Rightarrow x \geq a$

$\because \quad x > 0 \;\therefore\; a \leq 0$

23. (a) We have $u = 2^{\cos^2 x}$ and $v = \cos^2 x$

$\therefore \; \frac{du}{dx} = 2^{\cos^2 x} \cdot \log 2 \cdot (-2\cos x \sin x)$

$= -2\sin x \cos x \cdot \log 2 \cdot 2^{\cos^2 x}$

and $\frac{dv}{dx} = -2\sin x \cos x$

Derivative of $2^{\cos^2 x}$ w.r.t. $\cos^2 x$ is given by

$\frac{du/dx}{dv/dx} = \frac{-2\sin x \cdot \cos x \cdot \log 2 \cdot 2^{\cos^2 x}}{-2\sin x \cdot \cos x} = \log 2 \cdot 2^{\cos^2 x}$

OR

(b) We have, $\tan^{-1}(x^2 + y^2) = a^2$

Differentiating both sides w.r.t. x, we get

$\frac{1}{1 + (x^2 + y^2)^2}\left(2x + 2y\frac{dy}{dx}\right) = 0$

$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

24. We have, $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

As, $\sin^{-1}(\sin x) = x \;\forall\; x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore\ \sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{5}\right)\right) = \frac{2\pi}{5}$

25. Here, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

Thus, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)$

$= -2\hat{i} + 3\hat{j} + 7\hat{k}$

$\therefore$ Area of parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$= \frac{\sqrt{4+9+49}}{2} = \frac{\sqrt{62}}{2}$ units

26. (a) For the given lines,

$\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$

$= (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$

and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$

$= (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

Here, $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$.

$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$

$= 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$

Thus, $\Delta = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

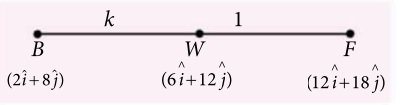
$= \left|\frac{(2\hat{i} - 4\hat{j} - 3\hat{k}) \cdot (\hat{j} - 4\hat{k})}{\sqrt{4+16+9}}\right| = \frac{8}{\sqrt{29}} \neq 0$

Hence, the given lines are skew lines $(\because\ \vec{b}_1 \neq k\vec{b}_2)$ and the shortest distance between lines is $\frac{8}{\sqrt{29}}$ units.

OR

(b) Given, $\vec{B} = 2\hat{i} + 8\hat{j}, \vec{W} = 6\hat{i} + 12\hat{j}, \vec{F} = 12\hat{i} + 18\hat{j}$

Let wicketkeeper divides the lines segment in $k : 1$ ratio.



Using section formula, we have

$6\hat{i} + 12\hat{j} = \frac{k(12\hat{i} + 18\hat{j}) + 1(2\hat{i} + 8\hat{j})}{k+1}$

$\Rightarrow\ 6(k+1)\hat{i} + 12(k+1)\hat{j} = (12k+2)\hat{i} + (18k+8)\hat{j}$

On comparing, we get $6(k+1) = 12k + 2$

$\Rightarrow\ 6k + 6 = 12k + 2$

$\Rightarrow\ 6k = 4 \Rightarrow\ k = \frac{2}{3}$

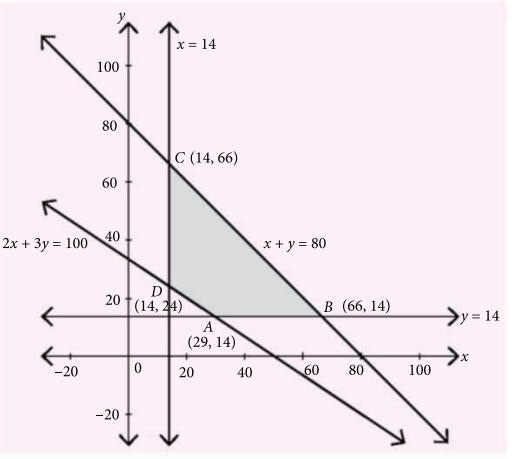
Hence, the line segment is divided in the ratio 2 : 3.

27. We have, Max $Z = 20x + 30y$

Subject to constraints, $x + y \leq 80$

$2x + 3y \geq 100$

$x \geq 14, y \geq 14$



From graph, $ABCD$, the feasible region and the corner points are $A(29, 14)$, $B(66, 14)$, $C(14, 66)$, $D(14, 24)$

The value of Z at these corner points are shown as :

Corner points	$Z = 20x + 30y$
$A(29, 14)$	1000
$B(66, 14)$	1740
$C(14, 66)$	2260 → Maximum
$D(14, 24)$	1000

Hence, maximum is at (14, 66) and maximum value is 2260.

28. Given, $\frac{dA}{dt} = 48 \text{ cm}^2/\text{s}$

Area of rectangle $A = lb = l^{3/2}$ $[\because\ l = b^2]$

Differentiating w.r.t. t, we get $\frac{dA}{dt} = \frac{3}{2} l^{1/2} \cdot \frac{dl}{dt}$

$\Rightarrow\ 48 = \frac{3}{2} l^{1/2} \cdot \frac{dl}{dt} \ \Rightarrow\ \frac{32}{\sqrt{l}} = \frac{dl}{dt}$

$\Rightarrow\ \frac{dl}{dt} = \frac{32}{4.5} = \frac{32 \times 2}{9} = \frac{64}{9}$ cm/s $[\because\ b = 4.5 \text{ cm}]$

29. (a) (i) $\sum P(x) = 1 \ \Rightarrow\ p + 2p + 3p + p = 1$

$\Rightarrow\ 7p = 1 \ \Rightarrow\ p = \frac{1}{7}$

(ii) Mean = $\Sigma XP(X) = 0\ (p) + 2(2p) + 4(3p) + 5p$

$= 21\text{p} = 21\left(\frac{1}{7}\right) = 3$

OR

(b) Total candidates = 3000

Number of females = $\frac{2}{3}(3000) = 2000$

Number of males = 1000

Let A be the event that a candidate will have a distinction in the written test.

E_1 be the event that a candidate is male.

E_2 be the event that a candidate is female.

Now, $P\left(\frac{A}{E_1}\right) = 0.4,\ P\left(\frac{A}{E_2}\right) = 0.35$

$\therefore\ P(E_1) = \frac{1000}{3000} = \frac{1}{3},\ P(E_2) = \frac{2000}{3000} = \frac{2}{3}$

Now, $P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)$

$= \frac{1}{3}\left(\frac{4}{10}\right) + \frac{2}{3}\left(\frac{35}{100}\right) = \frac{4}{30} + \frac{70}{300} = \frac{4}{30} + \frac{7}{30} = \frac{11}{30}$

30. (a) We have, $\int \frac{\cos 2x\ dx}{(\sin x + \cos x)^2}$

$= \int \frac{\cos 2x\ dx}{\sin^2 x + \cos^2 x + 2 \sin x \cos x} = \int \frac{\cos 2x\ dx}{1 + \sin 2x}$

Let $1 + \sin 2x = t \ \Rightarrow\ 2\cos 2x\ dx = dt$

$= \int \frac{dt}{2t} = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln|1 + \sin 2x| + C$

OR

(b) We have, $I = \int_0^{\pi/2} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x}\, dx$ ...(i)

$\Rightarrow\ I = \int_0^{\pi/2} \frac{5 \sin(\pi/2 - x) + 3 \cos(\pi/2 - x)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\, dx$

$\Rightarrow\ I = \int_0^{\pi/2} \frac{5 \cos x + 3 \sin x}{\sin x + \cos x}\, dx$ ...(ii)

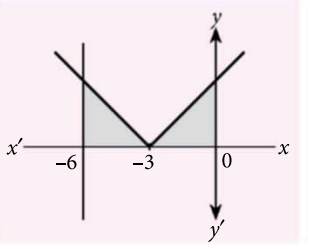
On adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{5(\sin x + \cos x) + 3(\sin x + \cos x)}{\sin x + \cos x}\, dx$

$I = \frac{8}{2} \int_0^{\pi/2} 1 \cdot dx = 4\,[x]_0^{\pi/2} = 4\left(\frac{\pi}{2} - 0\right)$

$\Rightarrow\ I = 2\pi$

31. Given, $y = |x + 3| = \begin{cases} -(x+3), & x \le -3 \\ x + 3, & x > -3 \end{cases}$



$\therefore$ Required Area $= \int_{-6}^{-3} (-x - 3)\, dx + \int_{-3}^{0} (x + 3)\, dx$

$= \left[\frac{-x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$

$= \left(\frac{-9}{2} + 9\right) - \left(\frac{-36}{2} + 18\right) + 0 - \left(\frac{9}{2} - 9\right)$

$= \frac{9}{2} + \frac{9}{2} = 9$ sq. units.

32. (a) Let $u = \tan^{-1} \frac{\sqrt{1 - x^2}}{x}$ ...(i)

Putting $x = \cos\theta$ in (i), we get

$u = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta = \cos^{-1} x$

$\Rightarrow\ \frac{du}{dx} = \frac{-1}{\sqrt{1 - x^2}}$ and $v = \cos^{-1}(2x\sqrt{1 - x^2})$ ...(ii)

Putting $x = \cos\theta$ in (ii), we get

$v = \cos^{-1}(2 \cos\theta \sin\theta) = \cos^{-1}(\sin 2\theta)$

$$= \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right) \quad = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\cos^{-1} x$$

$$\frac{dv}{dx} = \frac{-2(-1)}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}; \quad \frac{du}{dv} = \frac{du/dx}{dv/dx} = -\frac{1}{2}$$

OR

(b) We have, $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$ ...(i)

Let $v = x^{\tan x}$

Taking log on both sides, we get $\log v = \tan x \cdot \log x$

Differentiating w.r.t x, we get

$$\frac{1}{v}\cdot\frac{dv}{dx} = \log x \cdot \sec^2 x + \frac{\tan x}{x}$$

$$\frac{dv}{dx} = v\left(\log x \sec^2 x + \frac{\tan x}{x}\right)$$

$$= x^{\tan x}\left(\log x \sec^2 x + \frac{\tan x}{x}\right) \quad ...(ii)$$

Now, differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{dv}{dx} + \frac{1}{2}\left(\frac{1}{2}\right)\frac{(2x)}{\sqrt{x^2+1}}$$

$$\frac{dy}{dx} = x^{\tan x}\left(\log x \sec^2 x + \frac{\tan x}{x}\right) + \frac{x}{2\sqrt{x^2+1}}$$

[From (ii)]

33. Given, $f(x) = 2x^3 - 15x^2 + 36 + 1$

Differentiating on both sides, w.r.t. x, we get

$f'(x) = 6x^2 - 30x + 36$

For critical points, $f'(x) = 0$

$\Rightarrow \quad 6x^2 - 30x + 36 = 0 \Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow \quad (x-2)(x-3) = 0 \Rightarrow x = 2, 3$

We have been given interval [1, 5]

Hence, calculating $f(x)$ at 1, 2, 3 and 5

$f(1) = 2(1)^3 - 15(1)^2 + 36(1) + 1 = 2 - 15 + 36 + 1 = 24$

$f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 1 = 16 - 60 + 72 + 1 = 29$

$f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1 = 54 - 135 + 108 + 1 = 28$

$f(5) = 2(5)^3 - 15(5)^2 + 36(5) + 1 = 250 - 375 + 180 + 1 = 56$

Hence, absolute maximum value is 56 at $x = 5$ and absolute minimum value is 24 at $x = 1$.

34. Let x, y and z be the number of students in Sports, Music and Drama club.

$x = y + z \Rightarrow x - y - z = 0$ ...(i)

$y = \frac{1}{2}x + 20 \Rightarrow -\frac{1}{2}x + y = 20$ ...(ii)

$x + y + z = 180$ ...(iii)

Now, $A = \begin{bmatrix} 1 & -1 & -1 \\ -\frac{1}{2} & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow \quad |A| = 1(1-0) + 1\left(-\frac{1}{2} - 0\right) - 1\left(-\frac{1}{2} - 1\right) = 1 - \frac{1}{2} + \frac{1}{2} + 1$

$= 2 \neq 0$

$\therefore \quad A^{-1}$ exists.

$$Adj\ A = \begin{bmatrix} 1 & 0 & 1 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ -\frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{Adj\ A}{|A|} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ -\frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{1}{4} \\ -\frac{3}{4} & -1 & \frac{1}{4} \end{bmatrix}$$

The system of equations can be written as $AX = B$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{1}{4} \\ -\frac{3}{4} & -1 & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 0 \\ 20 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$$

$\therefore \quad x = 90$, $y = 65$ and $z = 25$

The number of students in the sports club is 90, in the music club is 65 and in the Drama club is 25.

35. (a) $A(1, 6, 3)$ is the given point and let L be the foot of perpendicular from A to the given line.

The coordinates of a general point on the given line are

$$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

i.e., $x = \lambda$; $y = 2\lambda + 1$ and $z = 3\lambda + 2$

If the coordinates of L are $(\lambda, 2\lambda + 1, 3\lambda + 2)$, then the direction ratios of AL are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$

Direction ratios of the given line are 1, 2, 3

Since, AL is perpendicular to the given line.

$(\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$

$\Rightarrow \quad \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0 \quad \Rightarrow \quad \lambda = 1$

Hence, coordinates of L are (1, 3, 5).

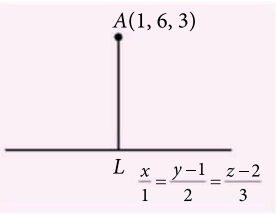
$A'(x, y, z)$ be the image of $A(1, 6, 3)$

The L is the mid-point of AA'.

Therefore, $\frac{x+1}{2}=1 \Rightarrow x=1$;

$\frac{y+6}{2}=3 \Rightarrow y=0$

and $\frac{z+3}{2}=5 \Rightarrow z=7$



Hence, the required image is $A'(1, 0, 7)$.

So, equation of AA' : $x-1=\frac{y-6}{-6}=\frac{z-3}{4}$

OR

(b) Given, $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ ...(i)

Coordinates of a general point P on line (i) are

$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$

$\Rightarrow x=\lambda-5, y=4\lambda-3$ and $z=-9\lambda+6$

Distance between $P(\lambda-5, 4\lambda-3, -9\lambda+6)$ and $Q(2\ 4, -1)$ is 7 units.

$\Rightarrow \sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$

$\Rightarrow \lambda^2+49-14\lambda+16\lambda^2+49-56\lambda+81\lambda^2+49-126\lambda=49$

$\Rightarrow 98\lambda^2-196\lambda+98=0 \quad \Rightarrow \lambda^2-2\lambda+1=0$

$\Rightarrow (\lambda-1)^2=0 \Rightarrow \lambda=1$

Hence, $P \equiv (-4, 1, -3)$

Equation of line joining P and Q is

$\frac{x-(-4)}{2-(-4)}=\frac{y-1}{4-1}=\frac{z-(-3)}{-1-(-3)}$

$\Rightarrow \frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$

36. Let E_1 denotes the interest of fixed rate type;
E_2 denotes the interest of floating rate type;
E_3 denotes the interest of variable rate type and A denotes the defaults on loan

$P(E_1)=\frac{10}{100}=0.1,\ P(E_2)=\frac{20}{100}=0.2,\ P(E_3)=\frac{70}{100}=0.7,$

$P\left(\frac{A}{E_1}\right)=\frac{5}{100}=0.05, P\left(\frac{A}{E_2}\right)=\frac{3}{100}=0.03$ and

$P\left(\frac{A}{E_3}\right)=\frac{1}{100}=0.01$

(i) $P(A)=P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)$

$= 0.1 \times 0.05 + 0.2 \times 0.03 + 0.7 \times 0.01 = 0.018 = \frac{9}{500}$

(ii) $P\left(\frac{E_3}{A}\right)=\frac{P(E_3)\cdot P\left(\frac{A}{E_3}\right)}{P(A)}$

$=\frac{0.7\times0.01}{0.018}=\frac{0.007}{0.018}=0.388=\frac{7}{18}$

37. Given, the total length of the boundary material is 300 metres.

The length of side perpendicular to the partition is x metres.

The length of side parallel to the partition is y metres.

(i) The total length of the boundary material is
$2x + 3y = 300$

(ii) Area of the solar panel is, $A = xy$ ...(i)

Since, $2x + 3y = 300$

$\Rightarrow 3y = 300 - 2x \quad \Rightarrow \quad y=\frac{300-2x}{3}$

Put y in equation (i), we get

$\Rightarrow A(x)=x\left[\frac{300-2x}{3}\right]=100x-\frac{2}{3}x^2$

(iii) (a) We have, $A(x)=100x-\frac{2}{3}x^2$

Differentiate both sides, w.r.t., x, we get

$\Rightarrow A'(x)=100-\frac{4}{3}x$

For critical point, $A'(x) = 0$

$\Rightarrow 100-\frac{4}{3}x=0 \quad \Rightarrow x=\frac{300}{4}=75$

Again, $A''(x)=\frac{-4}{3}$

Now, $A''(x)\big|_{x=75}=\frac{-4}{3}<0$ *i.e.*, $x = 75$ is a point of maxima.

$\therefore$ Maximum area $= A(75) = 100(75) - \frac{2}{3}(75)^2$

$= 7500 - 3750 = 3750\ m^2$

OR

(iii) (b) We have, $A = xy$ ...(i)

Also, perimeter $= 2x + 3y = 300 \Rightarrow 2x = 300 - 3y$

$\Rightarrow x=\left(150-\frac{3y}{2}\right)$

Put the value of x in equation (i), we get

$\therefore A(y)=\left(150-\frac{3y}{2}\right)y \qquad A(y)=150y-\frac{3y^2}{2}$

$\therefore A'(y) = 150 - 3y$

For critical points, $\therefore \quad 150 - 3y = 0$

$\Rightarrow 3y = 150 \Rightarrow y = 50$

Since, $A'(y)$ changes sign from positive to negative at y = 50. This point is a point of maxima.

$\therefore A(50)=\left(150-\frac{3\times50}{2}\right)\times50$ [From (i)]

$= 3750\ m^2$

38. (i) The relations R_1, R_2 and R_3 are not reflexive as $(1, 1), (2, 2), (3, 3) \notin R_1$, R_2 and R_3

The relation R_4 is not symmetric as $(1, 2) \in R_4$, but $(2, 1) \notin R_4$

Clearly, the relation R_4 is transitive

$\therefore$ The relation R_4 is reflexive and transitive but not symmetric.

(ii) The relation R_5 is reflexive and symmetric.

The relation R_5 is not transitive as $(3, 2)$ and $(2, 1) \in R_5$ but $(3, 1) \notin R_5$

$\therefore$ The relation R_5 is both reflexive and symmetric but not transitive.

(iii) (a) The relation R_1 is symmetric but neither reflexive nor transitive as

- $(2, 3), (3, 2) \in R_1$, therefore symmetric
- $(1, 1), (2, 2), (3, 3) \notin R_1$, therefore not reflexive
- $(2, 3)$ and $(3, 2) \in R_1$, but $(2, 2) \notin R_1$, therefore not transitive.

The relation R_2 is not symmetric as $(1, 2) \in R_2$ but $(2, 1) \notin R_2$

The relation R_3 is symmetric, but neither reflexive nor transitive as

- $(1, 2)$ and $(2, 1) \in R_3$, therefore symmetric
- $(2, 2)$ and $(3, 3) \notin R_3$, therefore not reflexive
- $(2, 1)$ and $(1, 2) \in R_3$ but $(2, 2) \notin R_3$, therefore not transitive.

The relation R_4 and R_5 are reflexive as $(1, 1)$, $(2, 2)$ and $(3, 3) \in R_4$ and R_5.

Hence, the relation R_1 and R_3 are symmetric but neither reflexive nor transitive.

OR

(iii) (b) The relation R_2 is not reflexive. To make R_2 as reflexive, (1, 1) (2, 2) and (3, 3) must be added to relation R_2 to make it reflexive.

The relation R_2 is also not symmetric. To make it symmetric, (2, 1), (3, 1), (2, 3) must be added.

Hence, to make the relation R_2 equivalence, 6 pairs must be added to R_2., *i.e.*, (1, 1), (2, 2), (3, 3), (2, 1), (3, 1), (2, 3).

PUBLIC NOTICE CUET (UG) 2025

Details for Common University Entrance Test (UG) - 2025:

1. Common University Entrance Test (CUET (UG) – 2025) will be conducted in 13 mediums across India for admission into the Undergraduate Programmes for all the Central Universities (CUs) and participating Universities in India for the academic year 2025–26.
2. Candidates may choose upto a maximum of five (05) subjects including languages and General Aptitude Test.
3. Broad features of CUET (UG) - 2025 are as follows:

Registration	Registration will be online at https://cuet.nta.nic.in
Total subjects	37 (13 languages + 23 domain-specific subjects + 01 General Aptitude Test).
Medium	13 Indian languages (Assamese, Bengali, English, Gujarati, Hindi, Kannada, Malayalam, Marathi, Punjabi, Odia, Tamil, Telugu, and Urdu)
Syllabus	Language Subjects: Language to be tested through Reading Comprehension (based on different types of passages–Factual, Literary, and Narrative), Literary Aptitude and Vocabulary
	Domain Subjects: As per NCERT syllabus
	General Aptitude Test: General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning (Simple application of basic mathematical concepts arithmetic/algebra geometry/mensuration/statistics), Logical and Analytical Reasoning
Choice of Test Paper	Candidates may choose upto a maximum of five (05) subjects including languages and General Aptitude Test irrespective of the subjects opted in class XII.*
Marking Scheme	Correct Answer: 05 marks (Five) There will be negative marking for each incorrect answer of 01 (one) mark.
Date of Examination	Between 08 May and 01 June 2025 (Tentative)
Mode of the Exam	Computer Based Test (CBT) mode
Total Questions	50 questions for each test papers. All Question are compulsory.
Duration	Duration for each test paper would be 60 minutes
Shift	The examination will be conducted on multiple shifts, depending on the number of Candidates and Subject choices.

*Candidates may select the subjects based on the eligibility criteria for a programme of a desired university

Website(s) https://cuet.nta.nic.in

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

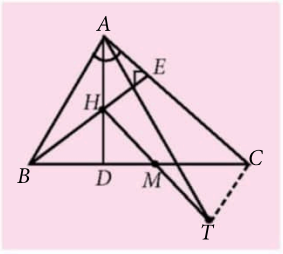
1. ABC is an acute angle triangle, $\angle A = 30°$. H is the orthocentre and M is the midpoint of BC. On the line HM a point T is taken such that $HM = MT$. If $BC = 4$ cm, then find the length of AT.

(Yana, Haryana)

Ans. ΔBMH and ΔCMT are similar.

$\therefore \quad \angle MBH = \angle MCT$

$\Rightarrow \quad BH \| CT$, $CT \perp AC$ and $CT = BH$



Now, $AT^2 = TC^2 + AC^2$

$= BH^2 + AC^2$

$= BD^2 \operatorname{cosec}^2 C + AD^2 \operatorname{cosec}^2 C$

$$= \frac{AB^2}{\sin^2 C} = \frac{a^2}{\sin^2 A} = 16 \times 4 = 64$$

$\therefore \quad AT = 8$

2. Little Sam is a unique child and his math marks show it. On four tests this year his scores out of 100 were all two-digit numbers made up of eight different non-zero digits. Also, the average of these scores is the same as the average if each score is reversed and this average is an integer none of whose digits is equal to any of the digits in the scores. What is Sam's average?

(Vivan, Mumbai)

Ans. Let the four marks be $10a + b$, $10c + d$, $10e + f$ and $10g + h$.

Then, $10a + b + 10c + d + 10e + f + 10g + h$

$= 10b + a + 10d + c + 10f + e + 10h + g$,

So, $9(a + c + e + g)$

$= 9(b + d + f + h)$

and $a + c + e + g$

$= b + d + f + h$. ...(i)

Therefore, the average of the four marks is

$$\frac{10a+b+10b+a+10c+d+10d+c+10e+f+10f+e+10g+h+10h+g}{4 \cdot 2}$$

$$= \frac{11(a+b+c+d+e+f+g+h)}{8} \quad \text{...(ii)}$$

The average must consist only of k's and 0's, the digits not equal to a, b, c, d, e, f, g or h.

Thus, from (ii), $\frac{11(1+2+\ldots+9-k)}{8} = \frac{11(45-k)}{8}$

and so $8 \div (45 - k)$. Since $k \le 9$, the only solution for k is $k = 5$. Therefore, Sam's average is $11(40)/8 = 55$.

3. Find the sum of all (distinct) complex values of c for which the polynomial

$f_c(x) = x^4 - (c^2 - 7c + 11)x^2 + (18 - 21c + 8c^2 - c^3)$

has strictly less than four distinct complex zeroes.

(Rashi, U.P.)

Ans. The polynomial $f_c(x)$ will fail to have four distinct complex zeros when the quadratic polynomial

$g_c(x) = x^2 - (c^2 - 7c + 11)x + (18 - 21c + 8c^2 - c^3)$

either has repeated roots or has 0 as a root.

Case 1: One of the roots of $g_c(x)$ is zero precisely when

$c^3 - 8c^2 + 21c - 18 = (c - 2)(c - 3)^2 = 0$

and so precisely when $c = 2, 3$

Case 2 : The roots of $g_c(x)$ are repeated when its discriminant is zero, so that

$(c^2 - 7c + 11)^2 - 4(18 - 21c + 8c^2 - c^3) = 0$

$\Rightarrow \quad c^4 - 10c^3 + 39c^2 - 70c + 49 = 0$

$\Rightarrow \quad (c^2 - 5c + 7)^2 = 0$

and this happens for two distinct values of c which sum to 5.

Thus the sum of the possible values of c is

$2 + 3 + 5 = 10$.

GK CORNER

Enhance Your General Knowledge with Current Updates!

SCIENCE AND TECHNOLOGY

1 In a ground breaking achievement, researchers from the Physical Research Laboratory (PRL) Ahmedabad, Panjab University Chandigarh, and Laboratory for Electro-Optics Systems at ISRO in Bengaluru created the first detailed map of the Moon's south pole. This significant milestone was made possible using data gathered by the Pragyan rover. The discovery offers new insights into the Moon's unexplored southern region, paving the way for future lunar exploration.

2 Recent research at the Wuhan Institute of Virology in China has identified a new bat coronavirus, HKU5-CoV-2, closely resembling SARS-CoV-2, the virus responsible for the COVID-19 pandemic. The study has raised questions about its potential for human transmission and the risk of future outbreaks. The discovery has sparked concerns about pandemic preparedness and zoonotic threats.

3 Recent advancements in medical research have led to a ground breaking in-utero treatment for Spinal Muscular Atrophy (SMA), a genetic disorder affecting motor neurons. This innovative approach marks a historic milestone as the first successful in-utero treatment for the condition. The development offers new hope for preventing the debilitating effects of SMA before birth.

4 The Indian National Space Promotion and Authorization Centre (IN-SPACe) has launched a ₹500-crore Technology Adoption Fund (TAF) to support space technology firms, especially startups and MSMEs. The initiative aims to foster innovation and create commercially viable products in India's space sector. The TAF is a strategic move to reduce India's dependence on imported space solutions and strengthen domestic research and development capabilities.

5 Microsoft has unveiled Majorana 1, a quantum chip designed to improve the reliability and scalability of qubits, marking a significant leap in quantum computing. The chip utilizes Majorana particles and topological conductors, a novel approach to qubit stability. With this advancement, Microsoft aims to develop a fully functional quantum computer by 2027-2029.

6 Recent studies have revealed crucial insights into solar coronal holes and their impact on space weather and the Indian summer monsoon. Coronal holes, dark regions observed in X–ray and extreme ultraviolet images of the Sun, characterised by open magnetic field lines, influence the interplanetary medium. Understanding them is key to comprehending space weather dynamics and its effects on Earth.

7 India's Deep Ocean Mission is making strides with the development of the Matsya-6000 submersible, spearheaded by the National Institute of Ocean Technology. Matsya–6000 has successfully completed wet testing at Chennai harbor. Designed to operate at depths of up to 6,000 meters, the submersible will aid in studying marine biodiversity and exploring ocean resources.

8 Recent breakthroughs in neutrino research have come from the KM3NeT (Cubic Kilometre Neutrino Telescope) Collaboration, which is building observatories deep beneath the Mediterranean Sea. The international team aims to detect neutrinos, subatomic particles that offer crucial insights into the universe's most dramatic events. A recent discovery involved a neutrino with unprecedented energy, potentially originating beyond the Milky Way galaxy.

9 Recent geological studies have found that the northern region of Iraq, especially around the Zagros Mountains, is undergoing slow but significant geological changes. Researchers have identified a sinking oceanic slab beneath the Earth's surface, gradually pulling the area downward. This phenomenon is a result of complex tectonic processes occuring over millions of years.

10 Prime Minister Narendra Modi accompanied by French President Emmanuel Macron, recently visited the International Thermonuclear Experimental Reactor (ITER) facility in France, to review the progress of the world's largest tokamak. The visit highlighted ITER's advancements in harnessing fusion energy, a key step towards carbon-free power generation. This initiative aims to demonstrate fusion as a viable and sustainable energy source for the future.

ATTENTION COACHING INSTITUTES : a great offer from MTG

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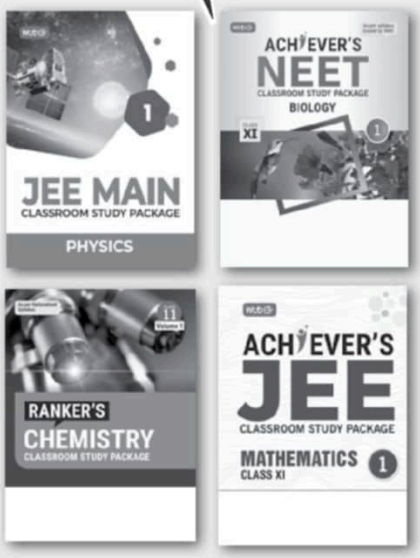

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Test Yourself!

1. What is the name of the newly identified bat coronavirus that resembles SARS-CoV-2?
   (a) HKU5-CoV-2
   (b) SARS-CoV-3
   (c) MERS-CoV-2
   (d) H1N1-CoV

2. What condition is being treated with the groundbreaking in-utero treatment discussed in recent medical research?
   (a) Cystic fibrosis
   (b) Spinal Muscular Atrophy (SMA)
   (c) Duchenne muscular dystrophy
   (d) Hemophilia

3. Which new quantum chip did Microsoft recently unveil?
   (a) Quantum-3000
   (b) Majorana 1
   (c) Qubit-7
   (d) Nano-Quantum 5

4. What is the main objective of the KM3NeT Collaboration in neutrino research?
   (a) To build the world's largest particle accelerator
   (b) To detect neutrinos for insights into the universe's dramatic events
   (c) To explore deep space beyond the Milky Way galaxy
   (d) To develop quantum computers

5. What geological phenomenon is affecting the northern region of Iraq, particularly around the Zagros Mountains?
   (a) Earthquake activity
   (b) A sinking oceanic slab pulling the area downward
   (c) Volcanic Eruptions
   (d) Sudden rise in sea levels

6. What significant achievement was made possible by the data gathered from the Pragyan rover?
   (a) Discovery of water ice on the Moon
   (b) Creation of the first detailed map of the Moon's South pole
   (c) Identification of a new lunar crater
   (d) Detection of seismic activity on the Moon

7. What does the ₹500-crore Technology Adoption Fund (TAF) aim to do for India's space sector?
   (a) Promote international space research partnerships
   (b) Support the development of new satellites
   (c) Foster innovation and create commercially viable products
   (d) Establish a new space station for lunar missions

8. What key feature characterizes solar coronal holes, as discussed in recent studies?
   (a) They are regions of intense solar flare activity
   (b) They are marked by open magnetic field lines
   (c) They emit large amounts of heat and energy
   (d) They have no effect on space weather

9. What is the purpose of India's Matsya-6000 submersible, developed as part of the Deep Ocean Mission?
   (a) To mine minerals from the ocean floor
   (b) To study marine biodiversity and explore ocean resources
   (c) To repair underwater cables
   (d) To detect underwater earthquakes

10. What is the main aim of the ITER facility, which Prime Minister Modi recently visited?
   (a) To build a new space station
   (b) To develop nuclear weapons
   (c) To harness fusion energy as a carbon-free power source
   (d) To create artificial intelligence for space exploration

Answer Key

1.	(e)	2.	(c)	3.	(b)	4.	(b)	5.	(a)
6.	(b)	7.	(c)	8.	(b)	9.	(b)	10.	(c)

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(b)	2.	(a)	3.	(c)	4.	(a)	5.	(b)
6.	(a)	7.	(a,c)	8.	(a,b,c)	9.	(a,b)	10.	(a,c)
11.	(a,b)	12.	(a)	13.	(a,c)	14.	(b)	15.	(c)
16.	(d)	17.	(3)	18.	(7)	19.	(5)	20.	(4)

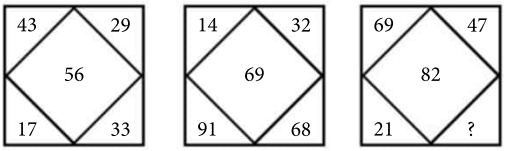
Brain Teaser

LOGICAL REASONING

For Various Competitive Exams

Direction Q.(1 and 2) : What will come in place of '?' in the given figures?

1.



(a) 55 (b) 82 (c) 79 (d) 60

2.

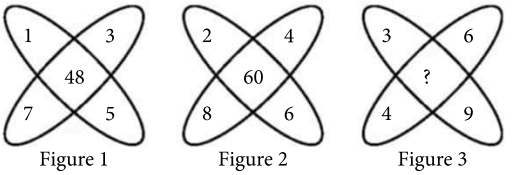


Figure 1 Figure 2 Figure 3

(a) 60 (b) 66 (c) 56 (d) 73

3. Fill in the blanks.

_ op _ mo _ n _ _ pnmop _

(a) mnpomn (b) mnompn
(c) mnpmon (d) mpnmop

Direction Q.(4 and 5) : Find the wrong term in the given series.

4. 150, 290, 560, 1120, 2140, 4230, 8400

(a) 2140 (b) 560
(c) 1120 (d) 4230

5. 950, 661, 436, 269, 146, 65, 16

(a) 436 (b) 65
(c) 269 (d) 661

Direction Q.(6 and 7) : Study the following information to answer the following questions:

In a certain code language,
'economy receiving very fast' is written as 'va jo ni pa';
'very essence of economy' is written as 'su pa lo jo';
'fast money in banks' is written as 'gy bt ks va' and
'of banks in industry' is written as 'ks dm bt su'.

6. What does 'su' stand for?

(a) economy (b) banks
(c) of (d) None of these

7. What would be the code for 'essence of money'?

(a) lo pa su (b) su gy jo
(c) bt va gy (d) lo su gy

8. A man showed a boy next to him and said, "he is the son of my wife's sister-in-law but I am the only boy child of my parents." How is my son related to him?

(a) Cousin (b) Brother
(c) Uncle (d) Nephew

Direction Q.(9 and 10) : Read the following information carefully and answer the questions given below:

Point H is 6 m towards the East of point G, point R is 8 m North of point G. Point Q is exactly midway between point R and point G. Point K is 10 m to the South of point Q. Point L is 3 m towards the East of point Q. Point S is exactly midway between point G and point H.

9. If a persons walks 4 m towards the South from point L, takes a right turn and walks for another 3 m, then which of the following points would he reach?

(a) Q (b) G (c) K (d) H

10. If a person walks 8 m towards North from Points S, then which of the following points would he cross and how far will he be from point R?

(a) G, 4 m (b) H, 3 m
(c) L, 6 m (d) L, 3 m

Direction Q.(11 and 12) : In the following questions, the symbols @, $, *, # and δ are used with the following meaning as given below:

'P $ Q' means 'P is not smaller than Q'.
'P # Q' means 'P is neither greater than nor equal to Q'.
'P * Q' means 'P is not greater than Q'.
'P @ Q' means 'P is neither smaller than nor equal to Q'.
'P δ Q' means 'P is neither greater than nor smaller than Q'.

Now, in each of the following questions assuming the given statements to be true, find which of the conclusions given below is/are definitely true?

11. **Statements:** H @ T, T # F, F δ E, E * V
Conclusions: I. V $ F II. E @ T
III. H @ V IV. T # V

(a) (I), (II) and (III) are true
(b) (I), (II) and (IV) are true
(c) (II), (III) and (IV) are true
(d) (I), (III) and (IV) are true

12. Statements: D # R, R * K, K @ F, F $ J

Conclusions: I. J # R II. J # K

III. R # F IV. K @ D

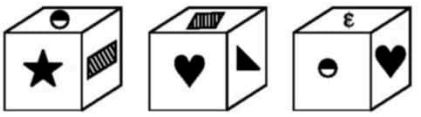
(a) (I), (II) and (III) are true

(b) (II), (III) and (IV) are true

(c) (I), (III) and (IV) are true

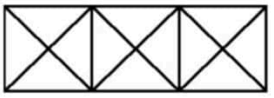
(d) (II) and (IV) are true

13. Which face will be opposite to the face containing the symbol ♥?



(a) (b) (c) (d)

14. How many triangles and squares are there in the given figure?



(a) 28 triangles & 5 squares

(b) 28 triangles & 4 squares

(c) 24 triangles & 4 squares

(d) 24 triangles & 5 squares

Direction Q.(15 and 16) : In each question given below, there are three statements followed by two conclusions numbered (I) and (II). You have to take the given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the three given statements, disregarding commonly known facts. Give your answer as

(a) Only conclusion (I) follows.

(b) Only conclusion (II) follows.

(c) Either (I) or (II) follows.

(d) Neither (I) nor (II) follows.

15. Statements:

All clouds are vapours.

No vapour is a gas.

All gases are rain.

Conclusions:

I. All rains being clouds is a possibility.

II. No cloud is a gas.

16. Statements:

All crops are fields.

No yield is a field.

All fields are harvests.

Conclusions:

I. No field is a crop.

II. All crops being harvest is a possibility.

Direction Q.(17 and 18) : Each of the questions below consists of a question and two statements numbered (I) and (II) given below it. You have to decide whether the data provided in the statements is sufficient to answer the question. Read both the statements and mark your answer as:

(a) if the data in statement (I) alone are sufficient to answer the question, while the data in statement (II) alone are not sufficient to answer the question.

(b) if the data in statement (II) alone are sufficient to answer the question, while the data in statement (I) alone are not sufficient to answer the question.

(c) if the data in both the statements (I) and (II) together are necessary to answer the question.

(d) if the data given in both the statements (I) and (II) together are not sufficient to answer the question.

17. Five persons P, Q, R, S, and T are sitting in a row. Q is between P and T. Who is sitting in the middle?

I. P is left of Q and right of S.

II. R is at the right end.

18. Has the price of silver gone up?

I. If the price of gold comes down, then the price of silver goes up.

II. The price of gold has gone up.

Direction Q.(19 and 20) : Study the following information carefully and answer the questions given below:

Six friends — Deepak, Varun, Anil, Nilesh, Rajesh and Siddharth are studying six different specialisations of engineering which are — metallurgy, telecommunication, software, mechanical, electrical and hardware not necessarily in the same order. Each one likes a different sport — hockey, cricket, swimming, football, badminton and tennis again not in the same order.

Nilesh is not studying hardware. Rajesh is studying software and likes hockey. Anil likes swimming and is not studying hardware. The one who likes football is studying electrical. Siddharth is studying mechanical and does not like tennis. The one who likes badminton is studying telecommunication. Deepak and Varun do not like badminton. Deepak does not like tennis.

19. Which sport does Deepak like?

(a) Football (b) Cricket

(c) Hockey (d) Cannot be determined

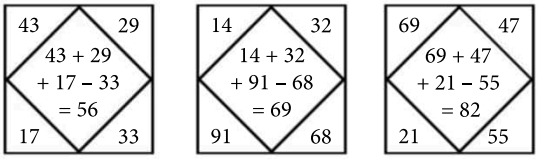
20. Which of the following person specialisation combination is correct according to the given information?

(a) Nilesh - Hardware (b) Varun - Electrical

(c) Anil - Metallurgy (d) Siddharth - Software

SOLUTIONS

1. (a) :



$\therefore$? = 55

2. (b) : Figure 1 : $3 \times (1 + 3 + 5 + 7) = 48$

Figure 2 : $3 \times (2 + 4 + 6 + 8) = 60$

Figure 3 : ? $= 3 \times (3 + 6 + 9 + 4) = 66$

3. (c)

4. (c) :

$$150 \xrightarrow{\times 2-10} 290 \xrightarrow{\times 2-20} 560 \xrightarrow{\times 2-30}$$

$$\boxed{1090} \xrightarrow{\times 2-40} 2140 \xrightarrow{\times 2-50} 4230 \xrightarrow{\times 2-60} 8400$$

$\therefore$ Wrong term is 1120

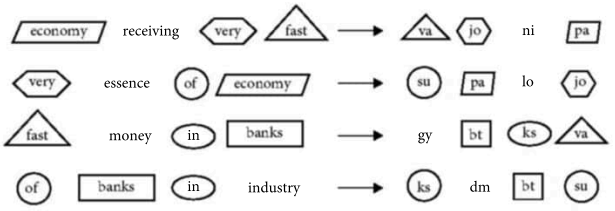
5. (c) :

$$950 \xrightarrow{-(17)^2} 661 \xrightarrow{-(15)^2} 436 \xrightarrow{-(13)^2}$$

$$\boxed{267} \xrightarrow{-(11)^2} 146 \xrightarrow{-(9)^2} 65 \xrightarrow{-(7)^2} 16$$

$\therefore$ Wrong term is 269

(6-7) :



6. (c) : The code for 'su' is 'of'.

7. (d) : The code for 'essence' is 'lo'.

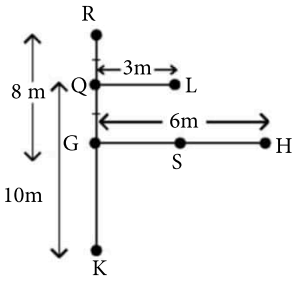
The code for 'of' is 'su'.

The code for 'money' is 'gy'.

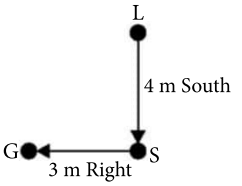
Hence, the code for 'essence of money' is lo su gy.

8. (a)

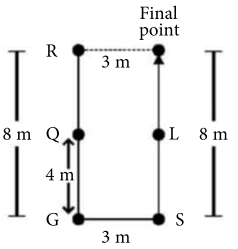
(9-10) :



9. (b) : Clearly, the person starts from L goes 4 m Southward and reaches point S, from where he takes right turn and walks 3 m to reach G.



10. (d) :



Clearly, man will cross L and at the final point, he will be at a distance of 3 m from R.

11. (b) : According to the question,

H @ T $\Rightarrow$ H > T

T # F $\Rightarrow$ T < F

F δ E $\Rightarrow$ F = E

E * V $\Rightarrow$ E $\leq$ V

Now, H > T < F = E $\leq$ V

Conclusions:

I. V \$ F $\Rightarrow$ V $\geq$ F (true)

II. E @ T $\Rightarrow$ E > T (true)

III. H @ V $\Rightarrow$ H > V (false)

IV. T # V $\Rightarrow$ T < V (true)

So, it is clear that conclusion (I), (II) and (IV) are true.

12. (d) : According to the question,

D # R $\Rightarrow$ D < R

R * K $\Rightarrow$ R $\leq$ K

K @ F $\Rightarrow$ K > F

F \$ J $\Rightarrow$ F $\geq$ J

Now, D < R $\leq$ K > F $\geq$ J

Conclusions:

I. J # R $\Rightarrow$ J < R (false)

II. J # K $\Rightarrow$ J < K (true)

III. R # F $\Rightarrow$ R < F (false)

IV. K @ D $\Rightarrow$ K > D (true)

So, it is clear that conclusion (II) and (IV) are true.

13. (a) : From the views it can be seen that opposite to ♥, the symbols , , , will not come. Hence, the correct answer is ★.

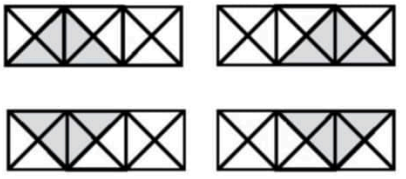
14. (a) :

There are 8 triangles *i.e.* 1, 2, 3, 4, 12, 23, 34, 41 and 1 square in such a figure.

Also, there are 3 such figures.

Hence, 3 × 8 = 24 triangles and 3 squares.

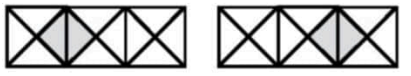
Also, 4 triangles are formed using combination of such figure, as shown below:



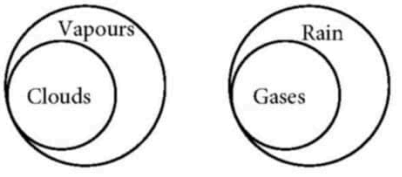
Hence, total 24 + 4 = 28 triangles.

Squares:

There are 2 more squares in given figure as shown below:

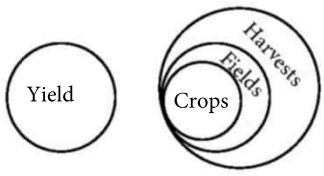


Hence, there are 3 + 2 = 5 squares.

15. (b) : 

As, some part of rain are gases and no vapour is a gas, thus conclusion (I) is invalid. As all clouds are vapours and no vapour is a gas, thus conclusion (II) is valid. Thus, only conclusion (II) follows.

16. (d) : As some part of field is crop, thus conclusion (I) is invalid. As all crops are fields and all fields are harvests. Thus conclusion (II) is invalid as it is not a possibility. Thus, none of the conclusion follows.



17. (c) : Using Statement (I) alone, we get:

S P Q T R

or

R S P Q T

So, either of P or Q can be in the middle seat.

Using Statement (II) alone, we get:

S P/T Q T/P R

or

P/T Q T/P S R

So, either of Q, T or P can be in the middle seat.

Now, by combining the two Statements, we get:

S P Q T R

So, Q is in the middle seat.

Hence, by combining the two statements, we get a unique answer. So, we will mark the answer as (c).

18. (d) : Using statement (I) alone, when price of gold goes down, then price of silver goes up. But price of silver goes up or not is not known.

Using statement (II) alone, price of gold gone up but nothing has been stated about price of silver and hence, we cannot find the answer.

As none of the statements alone is sufficient to answer the question, we combine the two statements:

When price of gold goes down, silver goes up; also gold goes up. But nothing has been given about what will happen to silver if gold goes up.

So, even by combining the two statements we cannot find the answer and hence the answer is (d).

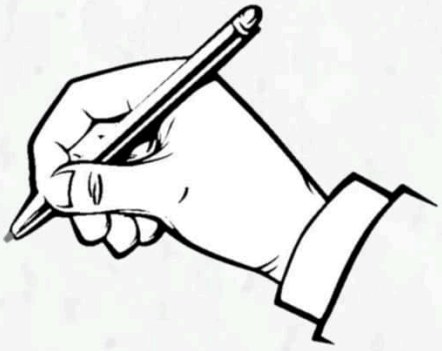
(19 – 20)

Friend	Specialisation	Sport
Deepak	Electrical	Football
Varun	Hardware	Tennis
Anil	Metallurgy	Swimming
Nilesh	Telecommunication	Badminton
Rajesh	Software	Hockey
Siddharth	Mechanical	Cricket

19. (a) : Deepak likes Football.

20. (c) : The combination Anil - Metallurgy is correct.

Infinite Learning



Unlocking the Mystery of Quadratic Equations: The Key to Solving Real-World Problems

Quadratic equations form the bedrock of modern algebra, often regarded as the foundation of higher mathematics. Although they may appear straightforward, quadratic equations possess a remarkable ability to model real-world phenomena, with applications spanning across fields such as engineering, economics, and even architecture and design.

In this article, we will delve into the fascinating world of quadratic equations, exploring their structure, solving techniques, and widespread applications.

What Are Quadratic Equations?

A **quadratic equation** is a second-degree polynomial equation of the form $ax^2 + bx + c = 0$

Here, $a(\neq 0)$ b and c are constants, and x represents the unknown variable. The term "quadratic" is derived from the Latin word "quadratus," meaning square, reflecting the squared term, x^2, which gives these equations their characteristic second-degree structure.

This squared term creates a parabola when graphed–a U-shaped curve that can either open upwards or downwards, depending on the value of a.

Solving Quadratic Equations

There are several methods for solving quadratic equations, each suited to different types of problems. The most common methods include:

1. **Factorisation:** This method involves expressing the quadratic equation as a product of two binomials. If factorisation is possible, equations can be solved easily and quickly.

   For example: . $x^2 + 5x + 6 = 0$

   $\Rightarrow \quad (x + 2)(x + 3) = 0.$

   Here, the solutions are $x = -2$ and $x = -3$..

2. **Completing the Square:** Sometimes factoring is tedious or not feasible, in that case, completing the square offers a more systematic approach. This technique involves manipulating the equation into a perfect square trinomial, allowing for the extraction of square roots. Consider the equation, $x^2 + 6x + 5 = 0$

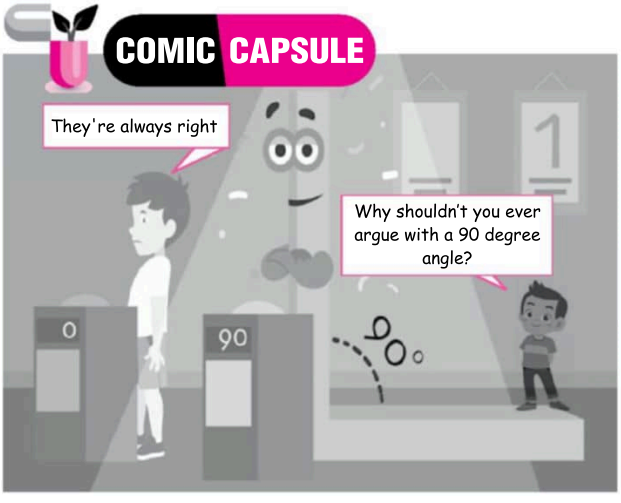
To complete the square, we first move the constant to the other side and add the square of half the coefficient of x to both sides:

$x^2 + 6x + 9 = -5 + 9 \Rightarrow (x + 3)^2 = 4 \Rightarrow x + 3 = \pm 2.$

Thus, $x = -1$ or $x = -5$.

3. **Quadratic Formula:** The quadratic formula is a universal method for solving any quadratic equation, regardless of whether it can be factored easily. This method is also known as Sridharacharya formula. The quadratic formula is given by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



The discriminant, $b^2 - 4ac$ inside the square root, reveals key information about nature of the roots.

- If the discriminant is positive, then there are two distinct real solutions.
- If the discriminant is zero, then there are two equal real solutions.
- If the discriminant is negative, then there are complex solutions (involving imaginary numbers).

For instance, solving $2x^2 + 3x - 5 = 0$ using the quadratic formula gives,

$$x = \frac{-3 \pm \sqrt{9 + 4(2)(5)}}{4} = \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$

Thus, the solutions are $x = 1$ and $x = -2.5$.

The Parabola: A Deeper Look

At the heart of every quadratic equation lies the **parabola**—a curve that holds key insights into the behaviour of the equation. The **vertex** of the parabola, the point where the curve reaches its peak or trough, is crucial for understanding the function's extremes. You can pinpoint this vertex using the formula,

$$x = -\frac{b}{2a}$$

The coordinates of the vertex reveal how the function behaves, indicating whether it's at its highest or lowest point. This not only tells us where the parabola changes its direction but also helps us understand the function's key characteristics.

Real-World Applications: More Than Just Math

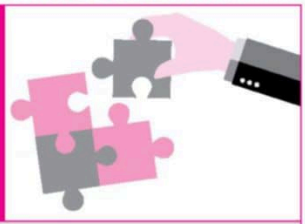
Quadratic equations are not just abstract concepts—they have a broad range of practical applications in the real world.

1. **Physics and Engineering:** Quadratic equations are used to model the motion of objects under gravity, such as the parabolic trajectory of a rocket or ball, helping predict when and where they will land. Engineers also apply quadratics in designing structures like bridges and arches, ensuring they are strong, stable, and efficient.
2. **Business and Economics:** In business, quadratic equations are used to model profit, cost and revenue functions, where the profit depends on the number of items produced or sold. For example, a company's profit could be represented by a quadratic function that maximizes at a certain production level with the **vertex** representing the optimal solution for maximum or minimum profit. By solving quadratic equations, businesses can determine the optimal production level or price point to maximize their earnings.
3. **Architecture and Design:** Parabolic arches in bridges or buildings are designed based on the properties of quadratic equations. These structures are strong and efficient because the shape distributes weight evenly. Engineers use quadratics to model and calculate the precise curvature of these arches.
4. **Computer Graphics:** Quadratic equations are used in computer graphics to render curves and surfaces. For instance, the shape of curved objects in video games, animation, or virtual reality environments can often be modelled using quadratic functions.

Conclusion

Quadratic equations transcend the boundaries of algebra—they are the gateway to solving real-world challenges. From optimizing business strategies and forecasting the trajectory of rockets to designing architectural marvels like bridges, mastering quadratics equips you to influence the world around you. Whether navigating straightforward problems or addressing intricate engineering dilemmas, quadratic equations form the cornerstone of both theoretical and applied mathematics. So, the next time you face a quadratic, remember: it's not merely an equation—it's a potent instrument for tackling the complex issues that propel progress in our world!

PUZZLE CORNER



MATHDOKU

Fill the grid with the numbers 1 to 8 such that each number appears only once in each row and column. The Grid is also divided in outlined regions called cages each with a given operator and target number. The numbers in the individual cells of a cage must produce that target number using the operator in a mathematical calculation.

180×			7–		1–	12×	
	5–		2–	6+		40×	
11+	1–				7–		
	6–		1–	6×		48×	
4–	2÷			15+		10×	
	15+			3–	25+		
22+	4÷				10×		
		16+				3÷	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. A sum of ₹ 11000 was taken as loan. This loan to be return in two equal annual instalments. If the rate of interest be 20% compounded annually, then value of each instalment will be
(a) ₹ 7500 (b) ₹ 7100 (c) ₹ 7000 (d) ₹ 7200

2. From an equilateral triangle of unit length, a new triangle is formed by joining the mid points, the process is repeated infinite times, what will be the perimeter of all triangles so formed including the original triangle?
(a) 8 units
(b) 6 units
(c) 4 units
(d) Cannot be determined

3. The volume of a hollow cube is $216x^3$, what is the surface area of the largest sphere which can be enclosed in it?
(a) $18\pi x^2$ sq.units (b) $24\pi x^2$ sq.units
(c) $27\pi x^2$ sq.units (d) $36\pi x^2$ sq.units

4. Two persons *A* and *B* walk around a circle 1500 metres in circumference. *A* walks at the rate of 150 m/min and *B* at the rate of 100 m/min. If they both start at the same point, same time and in same direction, then when will they first be together again at the starting point.
(a) After 30 min (b) After 45 min
(c) After $1\frac{1}{2}$ hour (d) After 2 hours

5. In a interschool chess championship every student had to play a game with every other student. It was observed that in 28 matches both players was boys and in 66 matches both were girls. What is the number of matches in which opposite sex are against each other?
(a) 528 (b) 462 (c) 196 (d) 96

6. A loan has to be returned in two equal annual payment, 16% p.a. compound interest being charged if each installment is of ₹ 1682, then which of the following is not true?
(a) The value of sum borrowed is ₹ 2700.
(b) Interest charged under installment scheme is ₹ 664.
(c) Total amount paid under installment scheme is ₹ 2700.
(d) The sum borrowed = present value of all installments.

7. There are two vessels *A* and *B*. Vessel *A* is containing 40 L of pure milk and vessel *B* is containing 22 litres of pure water. From vessel *A*, 8 L of milk is taken out and poured into the vessel *B*. Then 6 L of mixture (water and milk) from vessel *B* is taken out and poured into the vessel *A*. What is the respective ratio of quantity of pure milk in vessel *A* and quantity of pure water in vessel *B*?
(a) 21 : 13 (b) 14 : 5 (c) 21 : 11 (d) 24 : 13

Directions for questions 8 to 9: Answer the questions on the basis of the information given below:

Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Study both the statements and answer by selecting the choice as given below.
(a) If the data in statement I alone are sufficient to answer the question but the data in statement II alone are not sufficient to answer the question.
(b) If data in statement II individually sufficient to answer the question but data in statement I are not sufficient to answer the question.
(c) If data in both statement I and II together are necessary to answer the question.
(d) If the data even in both the statement I and II together are not sufficient to answer the question.

8. What is the man's speed in still water?
   I. The speed of stream is one-fourth the man's speed in still water.
   II. In the given time, the man can row thrice as far with the stream against it.

9. What is the speed of the stream?
   I. The ratio of the speed in upstream to the speed in downstream of boat is 3 : 4.
   II. The distance travelled upstream in 3 hours by the boat is more than the distance travelled by it downstream in 2 hours by 6 km.

10. There are 8 pipes attached with a tank out of which some are inlets and some are outlets. Every inlet can fill the tank in 12 hours and every outlet can empty the tank in 16 hours. When all the pipes are working together, the tank is filled up in $4\frac{4}{11}$ hours. If pair (a, b), denotes the number of inlets and outlets respectively, then which of the following is correct option?
   (a) (6, 2) (b) (4, 4) (c) (5, 3) (d) (3, 5)

11. Sanjay and Manoj undertaken to do a piece of work for ₹ 576. Sanjay alone can do the work in 8 days while Manoj can do it in 12 days. With Jatin, they together finish the work in 4 days. How should the money divided?
   (a) Sanjay = ₹ 192, Manoj = ₹ 96 and Jatin = ₹ 288
   (b) Sanjay = ₹ 288, Manoj = ₹ 192 and Jatin = ₹ 96
   (c) Sanjay = ₹ 96, Manoj = ₹ 288 and Jatin = ₹ 192
   (d) Sanjay = ₹ 96, Manoj = ₹ 192 and Jatin = ₹ 288

12. From amongst the available 15 cricket players, of whom 5 are bowlers, a team of 11 is to be selected. What is the probability that the selected team will have at least 3 bowlers?
   (a) $\frac{7}{13}$ (b) $\frac{5}{13}$ (c) $\frac{12}{13}$ (d) $\frac{9}{13}$

13. Sonu invested 10% more than the investment of Mona and Mona invested 10% less than the investment of Raghu. If the total investment of all the three persons is ₹ 5780, what is the investment of Raghu?
   (a) ₹ 1980 (b) ₹ 2000
   (c) ₹ 1800 (d) ₹ 3800

14. A person bought 8 quintals of rice for certain rupees. After a week, he sold 3 quintals of rice at 10% profit, 3 quintals at no profit-no loss and 2 quintals at 5% loss. What is the profit/loss in this transaction?
   (a) 2.5% loss (b) 5.0% loss
   (c) 3.5% profit (d) 2.5% profit

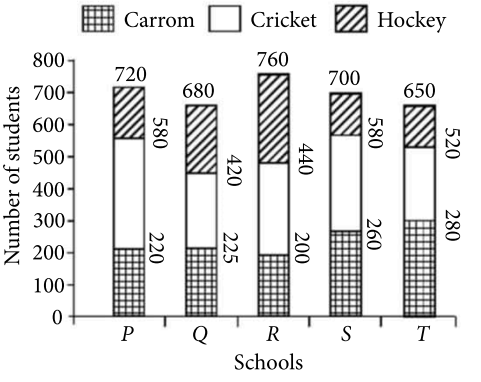
15. Due to 32% increase in the price of ghee a family reduces its consumption so that the price of ghee increase by 10% only. If before the increase of price the consumption of ghee was 10 kg, then find the consumption of ghee after increase of price?
   (a) $8\frac{1}{5}$kg (b) $8\frac{2}{5}$kg
   (c) $8\frac{2}{3}$kg (d) $8\frac{1}{3}$kg

16. If $x = (\sqrt{3}+1)^{-1/3}$, then $x^3 - \frac{1}{x^3}$ equals
   (a) $\frac{3-\sqrt{3}}{2}$ (b) $-\left(\frac{3+\sqrt{3}}{2}\right)$
   (c) $\frac{\sqrt{3}-3}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

17. If $\frac{x}{y} = \frac{3}{2}$, then $\frac{2x^2+3y^2}{3x^2-2y^2}$ equals
   (a) $\frac{4}{19}$ (b) $\frac{30}{19}$ (c) $\frac{10}{3}$ (d) $\frac{12}{5}$

Directions for questions 18 to 20 : Answer the questions on the basis of the information given below:

Number of students playing Carrom, Cricket and Hockey from different schools.



18. From school *R*, out of the students playing Carrom, 45% got selected for State level competition, out of which 30% further got selected for National level competition. From school *Q*, out of the students playing Carrom, 40% got selected for State level competition, out of which two-third further got

selected for National level competition. What is the total number of students playing Carrom from these two schools who got selected for National level competition?
(a) 80 (b) 85 (c) 87 (d) 90

19. Total number of students playing Carrom and Hockey together from school Q is what percent of the total number of students playing these two games together from school T?
(a) 118 (b) 115 (c) 95 (d) 85

20. If the number of students playing each game in school P is increased by 25% and the number of students playing each game in school T is decreased by 10%. What will be the difference between the number of students in schools P and T?
(a) 305 (b) 315 (c) 405 (d) 415

SOLUTIONS

1. (d) : Here, $n = 2$, $r = 20\%$
Let the value of each instalment be ₹ x, then

$$\frac{x}{\left(1+\frac{20}{100}\right)}+\frac{x}{\left(1+\frac{20}{100}\right)^2}=11000$$

$$\Rightarrow\ x\left(\frac{5}{6}+\frac{25}{36}\right)=11000\ \Rightarrow\ x\left(\frac{55}{36}\right)=11000$$

$$\Rightarrow\ x=\frac{11000\times 36}{55}=7200$$

2. (b) : Side of original triangle = 1 unit

Side of next (second) triangle $=\frac{1}{2}$ unit

Side of third triangle $=\frac{1}{4}$ unit

$\therefore$ Perimeter of original triangle = 3 units

$\therefore$ Perimeter of second triangle $=\frac{3}{2}$ units

$\therefore$ Perimeter of third triangle $=\frac{3}{4}$ units
and so on
$\therefore$ Perimeter of all the triangles

$$=3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}\ldots\infty \quad =3\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots\infty\right)$$

$$=3\left(\frac{1}{1-1/2}\right) \qquad \text{(Using G.P)}$$

$= 3 \times 2 = 6$ units

3. (d) : Given, Volume of cube $= 216x^3$
$\Rightarrow$ $(\text{edge})^3 = 216x^3 = (6x)^3$
$\Rightarrow$ Edge of cube $= 6x$ = Diameter of the sphere
$\therefore$ Radius of sphere $= 3x$
$\therefore$ Required area of sphere $= 4\pi\ (\text{radius})^2$
$= 4\pi \times 9x^2 = 36\pi x^2$ sq. units

4. (a) : Given, Length of circumference = 1500 m,
Speed of A = 150 m/min and speed of B = 100m/min
$\therefore$ Time taken by A to complete one revolution (round)

$$=\frac{1500}{150}=10 \text{ mins}$$

and time taken by B to complete one revolution

$$=\frac{1500}{100}=15 \text{ mins}$$

Thus, after every 10 min A will be at the starting point and after every 15 min B will be at the starting point. Hence, the time they meet again at starting point is the L.C.M. of 10 and 15 *i.e.*, 30 therefore after 30 mins *i.e.*, half an hour they meet again.

5. (d) : Let number of boys = m
$\therefore$ ${}^mC_2 = 28 \Rightarrow m(m-1) = 56$
or $m(m-1) = 8 \times (8-1)$
$\therefore$ $m = 8$
Again, let number of girls = n
$\therefore$ ${}^nC_2 = 66 \Rightarrow n(n-1) = 132$
$\Rightarrow$ $n(n-1) = 12 \times (12-1)$
$\Rightarrow$ $n = 12$
Therefore total number of participants = 12 + 8 = 20
$\therefore$ Total number of matches $={}^{20}C_2=\frac{20\times 19}{2\times 1}=190$
$\therefore$ Number of matches played between opposite sex
$= 190 - 28 - 66 = 96$

6. (c) : Given, Annual installment = ₹ 1682,
$R = 16\%$, $n = 2$
$\therefore$ Annual installment $=\dfrac{A}{\frac{100}{R}\left[1-\left(\frac{100}{100+R}\right)^n\right]}$

$$\Rightarrow\ 1682=\frac{A}{\left(\frac{100}{16}\right)\left[1-\left(\frac{100}{116}\right)^2\right]}\ \Rightarrow\ A = ₹\ 2700$$

(a) Value of sum borrowed = ₹ 2700
$\therefore$ Choice (a) is correct.
(b) Total amount paid under installment scheme

$= ₹\ 2 \times 1682 = ₹\ 3364$

So, interest paid $= 3364 - 2700 = ₹\ 664$

$\therefore$ Choice (b) is correct.

Choice (d) is well known fact therefore it is true.

Only choice (c) is not correct as the total amount paid under installment is $1682 \times 2 = ₹\ 3364$ but not ₹ 2700.

7. (c) : Quantity of pure milk in vessel $A = 40$ L

On taking out 8 L of pure milk from vessel A and taking out 6 L of mixture (milk and water) from vessel B and mixing in vessel A, then quantity of pure milk in the vessel A

$$= (40-8) + \frac{8}{22+8} \times 6 = 32 + \frac{8}{30} \times 6 = 32 + \frac{8}{5} = \frac{168}{5}\text{ L}$$

Similarly, on taking out 6 L mixture from vessel B, then quantity of pure water in vessel B left

$$= 22 - \frac{22}{(22+8)} \times 6 = 22 - \frac{22}{5} = \frac{88}{5}\text{ L}$$

$\therefore$ Required ratio of pure milk in vessel A and pure water in vessel B

$$= \frac{168}{5} : \frac{88}{5} = 168 : 88 = 21 : 11$$

8. (d) : Let the man's speed in still water be x km/h.

I. Speed of the stream $= \frac{x}{4}$ km/h

Speed of man in downstream

$$= \left(x + \frac{x}{4}\right)\text{ km/h} = \frac{5x}{4}\text{ km/h}$$

Speed of man in upstream

$$= \left(x - \frac{x}{4}\right)\text{ km/h} = \frac{3x}{4}\text{ km/h}$$

Let the given time be h hours, II gives,

$$3 \times \frac{5x}{4} \times h = \frac{3x}{4} \times h \qquad \text{(as distance covered relation)}$$

$\therefore$ I and II take together are not sufficient to give the answer.

9. (c) : I. The ratio of speeds upstream to speed downstream is 3 : 4.

Let the speed of upstream $= 3x$ km/h

and the speed of downstream $= 4x$ km/h

II. $(3 \times 3x) - (2 \times 4x) = 6$

$\Rightarrow 9x - 8x = 6 \Rightarrow x = 6$

$\therefore$ Speed of upstream = 18 km/h and speed of downstream = 24 km/h

$\therefore$ Speed of stream $= \frac{(24-18)}{2} = \frac{6}{2} = 3$ km/h

Thus, I and II jointly gives the answer therefore both are necessary to give the answer.

10. (c) : Let the number of outlets be x.

$\therefore$ Number of inlets $= 8 - x$

Time taken to fill the tank when all of them working together $= \frac{48}{11}$ hours

$\therefore$ Work done by all the pipes together in 1 hour $= 11/48$

According to problem, we have $\frac{8-x}{12} - \frac{x}{16} = \frac{11}{48}$

$\Rightarrow 4(8 - x) - 3x = 11 \quad \Rightarrow \quad x = 3$

$\therefore$ Number of outlets $(x) = 3$ and inlets $(8 - x) = 5$

$\therefore$ $(a, b) = (5, 3)$

11. (b) : One day work of Sanjay, Manoj and Jatin $= \frac{1}{4}$

(Sanjay + Manoj)'s one day work $= \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

$\therefore$ Jatin's one day work $= \frac{1}{4} - \frac{5}{24} = \frac{6-5}{24} = \frac{1}{24}$

$\therefore$ Sanjay's share : Manoj's share : Jatin's share

$$= \frac{1}{8} : \frac{1}{12} : \frac{1}{24} = 3 : 2 : 1$$

$\therefore$ Sanjay's share $= \frac{3}{6} \times 576 = ₹\ \frac{576}{2} = ₹\ 288$

Manoj's share $= \frac{2}{6} \times 576 = ₹\ \frac{576}{3} = ₹\ 192$

Jatin's share $= ₹\ \frac{1}{6} \times 576 = ₹\ 96$

12. (c) : Total players = 15

Total number of selected players = 11

$\therefore$ Total possible outcomes $= {}^{15}C_{11} = 1365$

Favourable number of cases

$= E(3 \text{ bowlers}) + E(4 \text{ bowlers}) + E(5 \text{ bowlers})$

($\because$ Total number of bowlers = 5)

$= ({}^5C_3 \times {}^{10}C_8) + ({}^5C_4 \times {}^{10}C_7) + ({}^5C_5 \times {}^{10}C_6)$

$= (10 \times 45) + (5 \times 120) + (1 \times 210) = 1260$

$\therefore$ $P(\text{at least 3 bowlers}) = \frac{1260}{1365} = \frac{12}{13}$

13. (b) : Let the share of Raghu be 100 units.

$\therefore$ Mona's share of investment is 10% less than Raghu = 90 units

and Sonu's share of investment 10% more than the share of Mona = 99 units

$\therefore$ Sonu's share : Mona's share : Raghu's share = 99 : 90 : 100

Total investment = ₹ 5780

$\therefore$ Investment of Raghu's $= \dfrac{5780 \times 100}{99+90+100}$

$= \dfrac{5780 \times 100}{289} =$ ₹ 2000

14. (d) : Let the cost price of 8 quintals of rice be ₹ $8x$ *i.e.* $(3x + 3x + 2x)$

$\therefore$ Cost price of 3 quintals of rice = ₹ $3x$

$\therefore$ S.P. of 3 quintals (at 10% profit) $=$ ₹ $\dfrac{3x \times 11}{10} =$ ₹ $\dfrac{33x}{10}$

S.P. of 3 quintals (at no profit no loss) = ₹ $3x$

S.P. of 2 quintals (at 5% loss) $=$ ₹ $\dfrac{2x \times 95}{100} =$ ₹ $\dfrac{19x}{10}$

$\therefore$ Total S.P. of 8 quintals of rice

$= ₹\left(\dfrac{33x}{10} + \dfrac{3x}{1} + \dfrac{19x}{10}\right) = ₹\dfrac{82x}{10}$

$\therefore$ Profit on 8 quintals rice $= ₹\left(\dfrac{82x}{10} - 8x\right) = ₹\dfrac{x}{5}$

$\therefore$ % Profit $= \dfrac{\frac{x}{5}}{8x} \times 100\% = \dfrac{100}{40}\% = \dfrac{5}{2}\%$

15. (d) : Let the original cost of ghee per kg be ₹ x.

Consumption of ghee before increase in price = 10 kg

$\therefore$ Expenses on ghee (before increase of price) = ₹ $(10x)$

As price of ghee is increased by 10 %

$\therefore$ Expense on ghee after increase in price

= 110% of $10x =$ ₹ $11x$

Now, new price of ghee = x + 32% of $x =$ ₹ $\dfrac{132x}{100}$

$\therefore$ New consumption $= \dfrac{\text{Total expenses}}{\text{Cost per kg}}$

$= \dfrac{11x}{\frac{132x}{100}} = \dfrac{11 \times 100}{132} = 8\dfrac{1}{3}$ kg

16. (b) : We have, $x = (\sqrt{3}+1)^{-1/3}$

$\Rightarrow x^3 = \dfrac{1}{\sqrt{3}+1} = \dfrac{\sqrt{3}-1}{2}$ ...(i)

and again $x^3 = \dfrac{1}{\sqrt{3}+1} \Rightarrow \dfrac{1}{x^3} = \sqrt{3}+1$ ...(ii)

From (i) and (ii), we get

$\therefore \quad x^3 - \dfrac{1}{x^3} = \dfrac{\sqrt{3}-1}{2} - (\sqrt{3}+1) = \dfrac{-\sqrt{3}-3}{2} = -\dfrac{(3+\sqrt{3})}{2}$

17. (b) : As, $\dfrac{x}{y} = \dfrac{3}{2} \Rightarrow \dfrac{x^2}{y^2} = \dfrac{9}{4}$

Now, $\dfrac{2x^2+3y^2}{3x^2-2y^2} = \dfrac{2\left(\frac{x^2}{y^2}\right)+3}{3\left(\frac{x^2}{y^2}\right)-2} = \dfrac{\frac{9}{2}+3}{\frac{27}{4}-2} = \dfrac{30}{19}$

18. (c) : From school *R*, Carrom players = 200

Selected players for state level competition

$= \dfrac{200 \times 45}{100} = 90$

Selected players for national level competition

$= \dfrac{90 \times 30}{100} = 27$

From school *Q*, Carrom players = 225

Selected players for state level competition

$= \dfrac{225 \times 40}{100} = 90$

Selected players for national level competition

$= \dfrac{90 \times 2}{3} = 60$

$\therefore$ Required number of students = 27 + 60 = 87

19. (a) : Total number of students playing Carrom and Hockey from school *Q* = 225 + 260 = 485

Total number of students playing these games from school *T* = 280 + 130 = 410

$\therefore$ Required percentage $= \dfrac{485}{410} \times 100 \approx 118\%$ (approximately)

20. (b) : New number of students in school *P*

$= \dfrac{720 \times 125}{100} = 900$

New number of students in school *T* $= \dfrac{650 \times 90}{100} = 585$

$\therefore$ Required difference = 900 – 585 = 315

10 BEST PROBLEMS *for* JEE Advanced 2025

1. Let $f: R - \left\{\frac{3}{2}\right\} \to R$, $f(x) = \frac{3x+5}{2x-3}$.

Let $f_2(x) = f(f(x)), f_3(x) = f(f_2(x)),\ldots,$

$f_n(x) = f(f_{n-1}(x))$. Then $f_{2008}(x) + f_{2009}(x) =$

(a) $\frac{2x^2+5}{2x-3}$ (b) $\frac{x^2+5}{2x-3}$

(c) $\frac{2x^2-5}{2x-3}$ (d) $\frac{x^2-5}{2x-3}$

2. If sum of x terms of a series is $S_x = \frac{1}{(2x+3)(2x+1)}$ whose r^{th} term is T_r. Then, $\sum_{r=1}^{n} \frac{1}{T_r}$ is equal to

(a) $\frac{1}{4}\sum(2r+1)(2r-1)(2r+3)$

(b) $-\frac{1}{4}\sum(2r+1)(2r-1)(2r+3)$

(c) $\sum(2r+1)(2r-1)(2r+3)$

(d) None of these

3. The number of solutions of the equation $\log_{(2x+3)}(6x^2+23x+21) + \log_{(3x+7)}(4x^2+12x+9) = 4$ is

(a) 0 (b) 1 (c) 2 (d) > 2

4. From the focus of the parabola $y^2 = 2px$ as centre, a circle is drawn so that a common chord of the curves is equidistant from the vertex and the focus. The radius of the circle is

(a) $\frac{p}{4}$ (b) $\frac{3p}{4}$ (c) $\frac{3p}{8}$ (d) $\frac{\sqrt{3}p}{8}$

5. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the $A_i's$ and exactly 9 of the $B_i's$. Then n is equal to

(a) 115 (b) 83

(c) 45 (d) None of these

6. If m and M are the least and greatest values of $(\cos^{-1} x)^2 + (\sin^{-1} x)^2$, then $\frac{M}{m} =$

(a) 10 (b) 5 (c) 4 (d) 2

7. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, where $pq \neq 0$, are bisected by the x-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$

(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

8. Let a be a complex number such that $|a| = 1$. If the equation $az^2 + z + 1 = 0$ has a pure imaginary root, then tan (arg a) =

(a) $\frac{\sqrt{5}-1}{2}$ (b) $\frac{\sqrt{5}+1}{2}$

(c) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (d) $\sqrt{\frac{\sqrt{5}+1}{2}}$

9. Two vertices of a triangle are (33, 26) and (–2, 5). If the orthocentre is (1, 2), then the circumradius is

(a) $\sqrt{17}$ (b) $3\sqrt{17}$ (c) $5\sqrt{17}$ (d) 20

10. The angle between the lines whose direction cosines are given by $l + m - 2n = 0$, $2mn - nl + 2lm = 0$ is

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

SOLUTIONS

1. (a): $f(x) = \frac{3x+5}{2x-3}$,

$f_2(x) = f(f(x))$

$= \frac{3f(x)+5}{2f(x)-3} = \frac{3\left(\frac{3x+5}{2x-3}\right)+5}{2\left(\frac{3x+5}{2x-3}\right)-3} = x$

$\therefore$ $f_2(x) = f_4(x) = \ldots = f_{2008}(x) = x$,

$f(x) = f_3(x) = \ldots = f_{2009}(x) = \frac{3x+5}{2x-3}$

$\therefore\ f_{2008}(x)+f_{2009}(x)=x+\dfrac{3x+5}{2x-3}=\dfrac{2x^2+5}{2x-3}$

2. (b): We have, $S_x=\dfrac{1}{(2x+3)(2x+1)}$

$\therefore\ T_x=S_x-S_{x-1}$

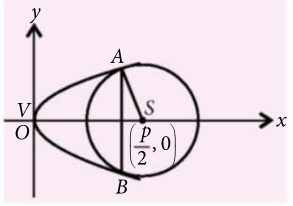
$\Rightarrow\ T_x=\dfrac{1}{(2x+3)(2x+1)}-\dfrac{1}{(2x+1)(2x-1)}$

$\Rightarrow\ T_x=\dfrac{-4}{(2x-1)(2x+1)(2x+3)}$

$\therefore\ \displaystyle\sum_{r=1}^{n}\frac{1}{T_r}=-\frac{1}{4}\sum_{r=1}^{n}(2r-1)(2r+1)(2r+3)$

3. (b): $6x^2+23x+21=(2x+3)(3x+7)$
and $4x^2+12x+9=(2x+3)^2$
$1+\log_{(2x+3)}(3x+7)+2\log_{(3x+7)}(2x+3)=4$
$\Rightarrow\ 1+t+\dfrac{2}{t}=4$ where $t=\log_{(2x+3)}(3x+7)$
$\Rightarrow\ t^2-3t+2=0\ \Rightarrow\ t=1,2$
If $t=1\Rightarrow\log_{(2x+3)}(3x+7)=1$
$\Rightarrow\ 3x+7=2x+3\Rightarrow x=-4,$
makes $2x+3<0$, $3x+7<0$, not valid.
$\therefore\ t=2\Rightarrow\log_{(2x+3)}(3x+7)=2$
$\Rightarrow\ 3x+7=(2x+3)^2\Rightarrow 4x^2+9x+2=0$
$\Rightarrow\ x=-2,\ -\dfrac{1}{4}$
$x=-2$ makes $2x+3<0$, not valid.
$\therefore\ x=-\dfrac{1}{4}$

4. (b): $y^2=2px\ \Rightarrow\ S\equiv\left(\dfrac{p}{2},0\right),\ V\equiv(0,0)$
The common chord is the perpendicular bisector of VS.
$\therefore$ The equation of AB is
$x=\dfrac{p}{4}$
It meets the parabola at
$A\left(\dfrac{p}{4},\dfrac{p}{\sqrt{2}}\right)$ since $y=\sqrt{2px}$



The radius of the circle is
$SA=\sqrt{\left(\dfrac{p}{2}-\dfrac{p}{4}\right)^2+\left(\dfrac{p}{\sqrt{2}}-0\right)^2}=\sqrt{\dfrac{p^2}{16}+\dfrac{p^2}{2}}=\dfrac{3p}{4}$

5. (c): $S=A_1\cup A_2\cup\ldots\cup A_{30}=B_1\cup B_2\cup\ldots\cup B_n$
Since each A_i contains 5 elements, number of elements in S(with repetitions) $=5\times30=150$
But each element of S appears exactly 10 times.
$\Rightarrow$ Number of distinct elements in $S=\dfrac{150}{10}=15$
Similarly by other union of B_i's, number of distinct elements in $S=\dfrac{3n}{9}$.
Now, $\dfrac{3n}{9}=15\Rightarrow n=45$

6. (a): $(\cos^{-1}x)^2+(\sin^{-1}x)^2$
$=(\cos^{-1}x+\sin^{-1}x)^2-2\sin^{-1}x\cos^{-1}x$
$=\dfrac{\pi^2}{4}-2\sin^{-1}x\left(\dfrac{\pi}{2}-\sin^{-1}x\right)$
$=2\left[(\sin^{-1}x)^2-\dfrac{\pi}{2}\sin^{-1}x+\dfrac{\pi^2}{8}\right]$
$=2\left[\left(\sin^{-1}x-\dfrac{\pi}{4}\right)^2+\dfrac{\pi^2}{16}\right]=f(x)$
$m=f\left(\dfrac{1}{\sqrt{2}}\right)=\dfrac{\pi^2}{8},$

SAMURAI SUDOKU

ANSWER · MARCH 2025

4	1	6	8	9	7	5	2	3
2	3	9	5	1	6	7	4	8
7	5	8	4	2	3	1	9	6
5	4	1	2	6	8	3	7	9
9	6	3	7	4	5	8	1	2
8	2	7	9	3	1	4	6	5
3	8	4	6	7	9	2	5	1
1	9	2	3	5	4	6	8	7
6	7	5	1	8	2	9	3	4

2	8	6	4	3	7	5	9	1
9	3	5	1	6	8	4	7	2
1	7	4	2	5	9	3	8	6
6	4	1	9	7	5	8	2	3
3	9	2	8	1	6	7	5	4
7	5	8	3	2	4	6	1	9
8	6	3	5	9	1	2	4	7
4	2	9	7	8	3	1	6	5
5	1	7	6	4	2	9	3	8

			8	1	3	5	6	9
			5	4	6	2	1	7
			7	9	2	3	4	8

7	4	2						
9	3	8						
1	5	6						

3	9	2	5	1	7	4	6	8
5	6	8	3	2	4	1	7	9
4	1	7	6	8	9	3	2	5
2	8	9	7	4	3	5	1	6
7	5	6	1	9	8	2	4	3
1	4	3	2	6	5	9	8	7
8	3	5	4	7	1	6	9	2
9	2	4	8	5	6	7	3	1
6	7	1	9	3	2	8	5	4

7	3	5	2	9	1	6	5	3
4	2	6	3	8	5	1	4	7
8	9	1	6	7	4	9	8	2
			1	6	9	5	3	8
			8	4	3	7	2	1
			5	2	7	4	9	6
			9	3	6	8	7	4
			4	5	2	3	1	9
			7	1	8	2	6	5

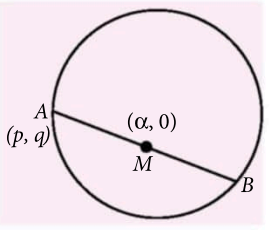
4	8	7						
6	9	2						
3	7	5						

7	6	8	4	3	1	2	9	5
5	4	3	9	2	6	8	7	1
1	2	9	8	5	7	4	3	6
6	1	7	5	3	8	9	4	2
8	5	4	7	9	2	6	1	3
9	3	2	1	6	4	7	5	8
3	9	6	8	7	4	1	5	2
4	5	2	3	1	9	7	6	8
7	1	8	2	6	5	3	4	9

$$M = f(-1) = 2\left[\left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$$

$$\frac{M}{m} = 10.$$

7. (d): $M(\alpha, 0)$ is on the x-axis.



M is the midpoint of AB.

$\therefore \quad B \equiv (2\alpha - p, -q)$

which lies on the circle

$x^2 + y^2 = px + qy$

$\therefore \quad (2\alpha - p)^2 + q^2$

$= p(2\alpha - p) - q^2 \Rightarrow 2\alpha^2 - 3p\alpha + p^2 + q^2 = 0$

As these are two chords, there are two values of α.

$\therefore$ Discriminant > 0.

$\Rightarrow \quad 9p^2 - 8(p^2 + q^2) > 0 \Rightarrow p^2 > 8q^2$

8. (d): $|a| = 1 \Rightarrow a = \cos\alpha + i \sin\alpha$

The equation is $(\cos\alpha + i \sin\alpha)\, z^2 + z + 1 = 0$

Now, $z = iy$ is a root, $y \in R$

$\Rightarrow \quad -(\cos\alpha + i \sin\alpha)\, y^2 + iy + 1 = 0$

Real parts $\Rightarrow y^2 \cos\alpha = 1$,

Imaginary parts $\Rightarrow y^2 \sin\alpha = y, y \neq 0 \Rightarrow y \sin\alpha = 1$

$$\Rightarrow \quad \frac{1}{\cos\alpha} = \frac{1}{\sin^2\alpha}$$

$\Rightarrow \quad \sin^2\alpha - \cos\alpha = 0, \cos^2\alpha + \cos\alpha - 1 = 0$

Solving, $\cos\alpha = \dfrac{\sqrt{5}-1}{2}$

$$\Rightarrow \quad \sec\alpha = \frac{2}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{2}$$

$$\tan^2\alpha = \sec^2\alpha - 1 = \left(\frac{\sqrt{5}+1}{2}\right)^2 - 1$$

$$= \frac{2\sqrt{5}+2}{4} = \frac{\sqrt{5}+1}{2} \quad \therefore \quad \tan\alpha = \sqrt{\frac{\sqrt{5}+1}{2}}$$

9. (c): Let $A \equiv (33, 26)$, $B \equiv (-2, 5)$, $C \equiv (\alpha, \beta)$

Here $O \equiv (1, 2)$, $OA \perp BC$

$$\Rightarrow \quad \left(\frac{24}{32}\right)\left(\frac{\beta - 5}{\alpha + 2}\right) = -1 \Rightarrow 4\alpha + 3\beta = 7 \qquad \ldots(i)$$

$$OB \perp CA \Rightarrow (-1)\left(\frac{\beta - 26}{\alpha - 33}\right) = -1 \Rightarrow \alpha - \beta = 7 \ldots(ii)$$

From (i), (ii) $\Rightarrow \quad \alpha = 4, \beta = -3$

$\therefore \quad C \equiv (4, -3)$

The centroid is $\left(\dfrac{33 - 2 + 4}{3}, \dfrac{26 + 5 - 3}{3}\right)$

or $\quad G \equiv \left(\dfrac{35}{3}, \dfrac{28}{3}\right)$

The circumcentre S divides OG in the ratio $3 : -1$.

$$\therefore \quad S \equiv \left(\frac{35 - 1}{2}, \frac{28 - 2}{2}\right) \Rightarrow S \equiv (17, 13)$$

The circumradius is $SA = \sqrt{(33 - 17)^2 + (26 - 13)^2}$

$$= \sqrt{16^2 + 13^2} = \sqrt{425} = 5\sqrt{17}$$

10. (d): Eliminating n, $(2m - l)\, n + 2lm = 0$,

$(2m - l)(l + m) + 4lm = 0$

$\Rightarrow \quad 2m^2 + 5ml - l^2 = 0$

If the roots are $\dfrac{m_1}{l_1}, \dfrac{m_2}{l_2}$ then $\dfrac{m_1 m_2}{l_1 l_2} = -\dfrac{1}{2}$ ...(i)

Eliminating m, $2(n + l)\, m - nl = 0$, $2(n + l)\,(2n - l) - nl = 0$

$4n^2 + nl - 2l^2 = 0$

If $\dfrac{n_1}{l_1}, \dfrac{n_2}{l_2}$ are the roots then $\dfrac{n_1 n_2}{l_1 l_2} = -\dfrac{1}{2}$ ...(ii)

On adding, (i) + (ii) $\Rightarrow \quad \dfrac{m_1 m_2 + n_1 n_2}{l_1 l_2} = -1$

$\Rightarrow \quad l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

The lines are perpendicular.

PUZZLE CORNER

SOLUTION - MARCH 2025

4– 7	5– 1	6	4+ 8	2	1– 5	4	16+ 3
3	14× 2	7	2+ 6	28× 1	4	5 5	8
20+ 6	640× 8	4	3	7	4× 2	1	5
1	4	5	14× 7	2– 8	9× 3	2	12+ 6
5	15+ 7	8	2	6	1	3	4
8	5 5	4× 1	4	90× 3	6	336× 7	2
15+ 4	1– 3	2	1	5	8	6	23+ 7
2	6	3	1– 5	4	7	8	1

Class XI

Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series-10 Statistics and Probability

Total Marks : 80 **Time taken :** 60 Min.

Only One Option Correct Type

1. If p, q are chosen randomly with replacement from the set $\{1, 2, 3, \ldots, 10\}$, the probability, that the roots of the equation $x^2 + px + q = 0$ are real, is
(a) 3/5 (b) 31/50 (c) 61/100 (d) 29/50

2. If 3 numbers are selected from the first 15 natural numbers, then the probability that the numbers are in A.P. is
(a) 7/65 (b) 9/65 (c) 8/65 (d) 6/65

3. If $a \in [-6, 12]$, then the probability that graph of $y = -x^2 + 2(a+4) - (3a+40)$ is strictly below x-axis is
(a) 2/3 (b) 1/3
(c) 1/2 (d) none of these

4. Two persons A and B are throwing an unbiased six faced die alternatively, with the condition that the person who throws 3 first wins the game. If A starts the game, the probabilities of A and B to win the same are respectively
(a) $\frac{6}{11}, \frac{5}{11}$ (b) $\frac{5}{11}, \frac{6}{11}$ (c) $\frac{8}{11}, \frac{3}{11}$ (d) $\frac{3}{11}, \frac{8}{11}$

5. The mean of the values 0, 1, 2, 3, ..., n with the corresponding weights ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ respectively, is
(a) $\frac{n+1}{2}$ (b) $n/2$ (c) $\frac{2^n - 1}{2}$ (d) $\frac{2^n + 1}{2}$

6. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data is
(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

One or More Than One Option(s) Correct Type

7. If A, B are two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$ then
(a) $P(A) + P(B) \leq \frac{11}{8}$ (b) $P(A) \cdot P(B) \leq \frac{3}{8}$
(c) $P(A) + P(B) \geq \frac{7}{8}$ (d) none of these

8. If A and B are two events such that $P(A) = 3/4$ and $P(B) = 5/8$, then
(a) $P(A \cup B) \geq 3/4$ (b) $P(A' \cap B) \leq 1/4$
(c) $3/5 \leq P(A \cap B) \leq 5/8$
(d) none of these

9. The probability that a 50 yr old man will be alive at 60 is 0.83 and the probability that a 45 yr old woman will be alive at 55 to 0.87. Then,
(a) the probability that both will be alive is 0.7221
(b) atleast one of them will alive is 0.9779
(c) atleast one of them will alive is 0.8230
(d) the probability that both will be alive is 0.6320

10. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the seconds. The chances of the events are
(a) $p_1 = \frac{1}{9}$ (b) $p_1 = \frac{1}{16}$ (c) $p_2 = \frac{1}{3}$ (d) $p_2 = \frac{1}{4}$

11. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then
(a) $\sigma^2 = \frac{f_1 x_1^2 + f_2 x_2^2}{f_1 + f_2} - \left(\frac{f_1 x_1 + f_2 x_2}{f_1 + f_2}\right)^2$

(b) $\sigma^2 = \dfrac{f_1 f_2}{(f_1+f_2)^2} - (x_1 - x_2)^2$

(c) $\sigma^2 = \dfrac{(x_1 - x_2)^2}{(f_1+f_2)^2}$ (d) none of these

12. A scientist is weighing each of the 20 fishes. Their mean weight were 20 g and a standard deviation is 1 g later it was found that measuring scale was misaligned and under reported every fish by 2 g. Then the correct mean and standard deviation are

(a) 18, 1 (b) 19, 1 (c) 1, 5 (d) 2, 5

13. Consider a set of $3n$ numbers having variance 4. In this set, the mean of first $2n$ numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first $2n$ numbers and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then

(a) $9k = 68$ (b) $9k = 66$

(c) $3k = 22.67$ (d) $3k = 23.33$

Comprehension Type

Paragraph for Q. No. 14 and 15

Box I contains three tickets bearing numbers 1, 2, 3; box II contains five tickets bearing numbers 1, 2, 3, 4, 5, and box III contains seven tickets bearing numbers 1, 2, 3, 4, 5, 6, 7. A tickets is drawn from each of the boxes. Let x_i be the number on the tickets drawn from the i^{th} box, $i = 1, 2, 3$.

14. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\dfrac{29}{105}$ (b) $\dfrac{53}{105}$ (c) $\dfrac{57}{105}$ (d) $\dfrac{1}{2}$

15. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\dfrac{9}{105}$ (b) $\dfrac{10}{105}$ (c) $\dfrac{11}{105}$ (d) $\dfrac{7}{105}$

Matrix Match Type

16. Consider the given data with frequency distribution.

x_i	2	7	10	9	4	3
f_i	3	1	2	5	3	2

Match the columns.

Column-I		Column-II	
(P)	The mean of above data is	(1)	3
(Q)	The median of above data is	(2)	6
(R)	The mean deviation about the mean of the above data is	(3)	9
(S)	The mean deviation about the median of the above data is	(4)	3.25

(a) (P) → (3); (Q) → (1); (R) → (4); (S) → (2)
(b) (P) → (1); (Q) → (2); (R) → (4); (S) → (3)
(c) (P) → (4); (Q) → (3); (R) → (2); (S) → (1)
(d) (P) → (2); (Q) → (3); (R) → (1); (S) → (1)

Numerical Answer Type

17. A clerk was asked to mail three report cards to three students. He addresses three envelopes but unfortunately paid no attention to which report card be put in which envelop. Then the favourable outcomes of exactly one of the students received his or own card is _____.

18. A and B throw a die. The probability that A's throw is not greater than B's is $\dfrac{p}{p+5}$. Then value of p is _____.

19. The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is _____.

20. If the mean is x and the standard deviation is y for the following data :

Age (years)	No. of teachers
25 – 30	30
30 – 35	23
35 – 45	20
40 – 45	14
45 – 50	10
50 – 55	3

Then the integer value of $\left[\dfrac{x}{y}\right]$ is _____.

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No. of questions correct .....
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< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

Class XII

Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series-10 Linear Programming and Probability

Total Marks : 80 **Time taken :** 60 Min.

Only One Option Correct Type

1. The minimum value of $z = 4x + 2y$ subject to $x + y \geq 8$, $4x + y \geq 12$, $x, y \geq 0$ is

(a) $\frac{56}{3}$ (b) 24 (c) 32 (d) 36

2. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items. A fan and sewing machine cost ₹ 360 and ₹ 240 respectively. He can sell a fan at a profit of ₹ 22 and sewing machine at a profit of ₹ 18. Assuming that he can sell whatever he buys, how many fans and sewing machines respectively he sell in order to maximize his profit?

(a) 8, 12 (b) 6, 10 (c) 10, 6 (d) 12, 8

3. The maximum value of $z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$ is

(a) 9 (b) 10 (c) $\frac{235}{19}$ (d) $\frac{205}{21}$

4. A student appears for tests I, II and III. The student is successful if he passes either in test I and II or test I and III. The probabilities of the student passing in tests I, II, III are p, q, $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$

(c) $p = 1, q = 0$ (d) $p = 1, q = \frac{1}{2}$

5. Two fair dice are rolled. Let X be the event that the first die shows an even number and Y be the event that the second die shows an odd number. Then X and Y are

(a) mutually exclusive

(b) independent and mutually exclusive

(c) dependent

(d) none of these

6. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively , is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, the probability that the original signal was green is

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$

(c) $\frac{12}{13}$ (d) $\frac{9}{20}$

One or More than One Option(s) Correct Type

7. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then

(a) E and F are mutually exclusive

(b) E and $\overline{F}$ are independent

(c) $\overline{E}$ and $\overline{F}$ are independent

(d) $P(E/F) + P(\overline{E}/F) = 1$

8. A fair coin is tossed n times. If X is the number of times heads occur and $P(X = 4)$, $P(X = 5)$, $P(X = 6)$ are in A.P., then n =

(a) 13 (b) 7 (c) 11 (d) 14

9. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$ then

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A' - B') = P(A') - P(B')$

(c) $P(A \cup B)' = P(A')\,P(B')$

(d) $P(A/B) = P(A)$

10. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

(b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$

(d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

11. If A and B are two independent events such that $P(A' \cap B) = 2/15$ and $P(A \cap B') = 1/6$, then $P(B)$ is

(a) 1/5 (b) 1/6 (c) 4/5 (d) 5/6

12. Three numbers are chosen at random without replacement from {1, 2, ..., 15}. Let E_1 be the event that minimum of the chosen numbers is 5 and E_2 their maximum is 10 then

(a) $P(E_1) = \frac{9}{91}$ (b) $P(E_2) = \frac{36}{455}$

(c) $P(E_1 \cap E_2) = \frac{4}{455}$ (d) $P(E_1 / E_2) = \frac{1}{9}$

13. An electric component manufactured by 'RASU Electronics' is tested for its defectiveness by a sophisticated testing device. Let A denote the event "the device is defective" and B the event "the testing device reveals the component to be defective". Suppose $P(A) = \alpha$ and $P(B/A) = P(B'/A') = 1 - \alpha$, where $0 < \alpha < 1$, then

(a) $P(B) = 2\alpha\,(1 - \alpha)$

(b) $P(A'/B) = 1/2$

(c) $P(B') = (1 - \alpha)^2 + \alpha^2$

(d) $P(A'/B') = [\alpha/(1 - \alpha)]^2$

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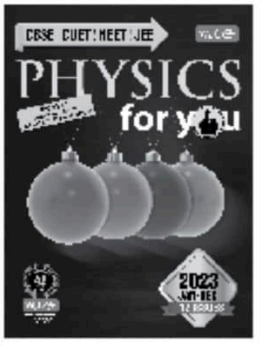

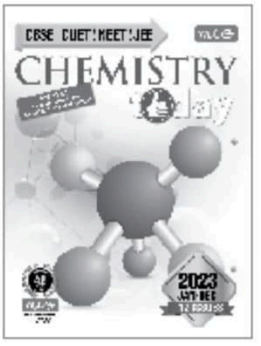

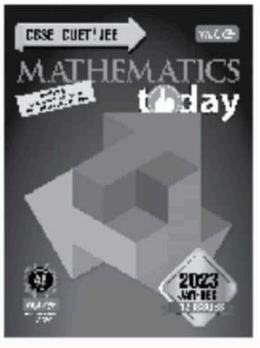

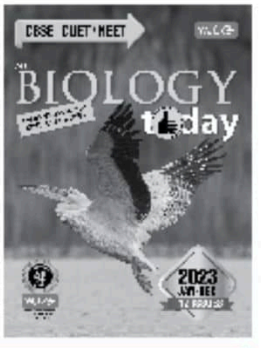

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Comprehension Type

Paragraph for Q. No. 14 and 15

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

14. The probability of the drawn ball from U_2 being white is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$

(c) $\frac{19}{30}$ (d) $\frac{11}{30}$

15. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$

(c) $\frac{15}{23}$ (d) $\frac{12}{23}$

Matrix Match Type

16. Match the following.

Column-I		Column-II	
(P)	The probability that length of a randomly selected chord of a circle lies between $\frac{1}{4}$ and $\frac{3}{4}$ of its diameter is	(1)	1/30
(Q)	Three numbers are selected from the set {1, 2, 3, …, 10}. The probability that they form a G.P. is	(2)	1/2
(R)	If x and y are selected from the set {1, 2, 3, …, 10}, the probability, that $x^3 + y^3$ is a multiple of 3, is	(3)	1/4
(S)	A card is lost from a pack of 52 playing cards. If a card is drawn from the remaining cards, the probability that it is a spade is	(4)	1/3

(a) (P) → (2); (Q) → (1); (R) → (4); (S) → (3)

(b) (P) → (1); (Q) → (3); (R) → (2); (S) → (4)

(c) (P) → (3); (Q) → (2); (R) → (4); (S) → (1)

(d) (P) → (2); (Q) → (3); (R) → (1); (S) → (4)

Numerical Answer Type

17. The minimum value of $z = 2x + 5y$ subject to $x + 2y \geq 10$, $3x + 4y \leq 24$, $x, y \geq 0$ is _____.

18. Let A and B be two events such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then

$P(B/A \cup B') = \frac{1}{n}$, where n is

19. Each of two boxes contains both black and white marbles, and the total number of marbles in the two boxes is 25. One marbles is taken out of each box. The probability that both marbles are black is 27/50. If the probability that both marbles are white is $\frac{m}{n}$, where m, n, are relatively prime positive integers, then m is _____.

20. A peon has eight master keys to open several new rooms. Only one master key will open any given room. If 40% of these rooms are usually left unlocked, then the probability that the peon gets into specific room if he selects three master keys at random before leaving the office is $\frac{5}{p}$ where p is _____.

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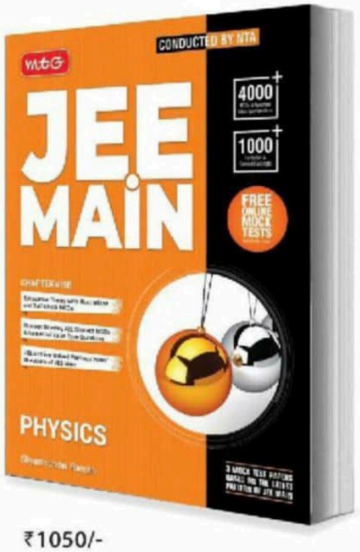

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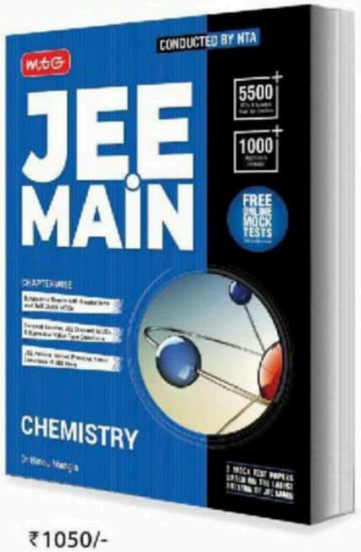

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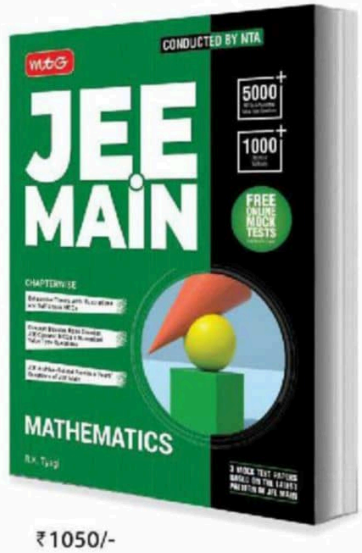

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